MONTHLY NOTICES

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Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

Zia Uddin Ahmad, D.Sc., Trinity College, Cambridge;
Daniel Buckney, 61 Strand, W.C., and 51 Highbury Park, N.; and
Robert James Wallace, Yerkes Observatory, Williams Bay, Wis., U.S.A.

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:

Major C. F. Close, C.M.G., R.E., Brompton Barracks, Chatham (proposed by Sir David Gill); and
Thomas Andrew Common, 63 Eaton Rise, Ealing, W. (proposed by W. H. M. Christie).

Eighty presents were announced as having been received since the last meeting, including, amongst others:

S. C. Chandler, Revision of Elements of third Catalogue of Variable Stars (presented by the author); Royal Observatory, Greenwich, Astrographic Catalogue, vol. i. (presented by the Observatory); Wislicenus, W. F., Astronomischer Jahresbericht, 1903 (presented by the author).

Note on the Instrumental Errors affecting Observations of the Moon. By F. W. Dyson, M.A., F.R.S.

1. In connection with Mr. Cowell's comparison of the observed positions of the Moon with the tabular positions Professor Turner contributed a note to the *Monthly Notices* of 1904 March in which he urged the necessity for more elaborate discussion of the instrumental errors to which observations of the Moon are liable, and suggested the possible utility of the observations made with the old altazimuth at Greenwich for this purpose. My reason for offering any remarks on the subject is that I have had opportunities of discussing the matter with Mr. Cowell and may be able to remove some misconceptions.

The ordinary sources of systematic errors in star catalogues such as a period in the adopted clock-star list, the adopted value of the equinox correction, flexure, the adopted system of refractions, &c., are of secondary importance in the discussion of short-period terms in the Moon's tables.

(a) Because the Moon moves so rapidly in both right ascension and declination.

(b) Because they are largely overwhelmed (α) by the personal errors in observations of limbs, (β) the errors arising from observations being sometimes made in a bright and sometimes in a dark sky, and (γ) the large accidental error arising from the limb being sometimes ill-defined and hazy.

2. The errors in transit-circle observations which are not peculiar to the Moon being capable of independent determination should be allowed for wherever their effect is likely to be appreciable. They will very rarely affect the terms of short period, but may be of importance in the terms of longer period.

3. Professor Turner's note as far as it criticised Mr. Cowell's procedure had reference to the errors peculiar to the Moon marked (b) in § 2. The errors of this class largely increase the accidental errors of an observation of the Moon over that of a star. Their effect will, however, be of an entirely accidental character on the coefficients of all short-period terms except those whose period is a lunation or an exact part of a lunation. Certainly in the long series of observations from 1750 to 1901 which Mr. Cowell is discussing we may expect them to be entirely eliminated.

4. Errors of this class have, as Mr. Cowell has been careful to note (*Monthly Notices*, lxiv. p. 94), a systematic effect on the parallactic inequality and on the variation. Mr. Cowell proposes to determine the coefficient of sin D and sin 2D separately for

each period of 400 days from 1750–1901, and to obtain from the accordance of the separate determinations a measure of their accuracy. In this long period there have been a good many observers and several instruments at Greenwich. Errors of a personal character which are systematic in a short series of observations may be fairly considered as accidental in a long one extending from 1750-1901. The detailed examination of these personal errors would probably increase the accordance of the results derived from the separate periods but not seriously affect the mean. It should be noted that such a discussion would not be possible or would, at least, be very rough till the tabular errors which Mr. Cowell is considering are allowed for. At the best personal errors are troublesome to investigate, and the results never free from the uncertainty that observers may have slowly changed their habits of observing.

5. Mr. Cowell's method of giving the coefficients as determined separately for each period of 400 days gives a ready method of comparing the results as given by the different instruments used since 1750. If discrepancies are shown larger than can be legitimately attributed to accidental error they will be put clearly in evidence and their amounts will be known. But there are, as Mr. Cowell remarks, no means available for determining the instrumental correction which may be required by any particular instrument to the Moon's diameter at different ages and different conditions. The agreement of the observed with the theoretical variation and of the solar parallax obtained from the observations 1847–1901 with values obtained in other ways shows that the errors of this class are not very large for the present transit circle, and affords ground for supposing that their effect on other terms, except the parallactic inequality, is extremely small.

6. With regard to the altazimuth observations it is no doubt true that the systematic errors to which they are liable are probably different from those inherent in meridian observations. Unfortunately they have a large accidental error, and quite probably large systematic errors, depending directly on the Moon's altitude, and thus indirectly on its age. The comparison of the observed with the tabular places might be analysed. This would have the advantage over Professor Turner's comparison with the transit circle that all the observations could be used. Whether the results would repay the labour is, I think, a very open question.

7. Professor Turner uses the altazimuth observations as a standard to compare the old transit circle in use from 1847–1851 with the present one, 1851–1861. This comparison is, it seems to me, made better *via* the more uniform tabular places, and can be taken from Mr. Cowell's paper (*Monthly Notices*, vol. lxiv. p. 526, col. 6). The periods 86, 87, 88 in Mr. Cowell's paper correspond to the years 1847–1851, and the periods 90–99 to the years 1851–1861. I extract the figures for these periods.

1847–1851. Period.	Coeff. of sin D.	Period.	1851–1861. Coeff. of sin D.	Period.	Coeff. of sin D.
	″		″		″
86	+0·2	90	−0·5	95	−0·9
87	−1·1	91	+0·5	96	−1·2
88	−0·1	92	−0·8	97	−1·3
		93	0·0	98	−0·9
		94	−0·5	99	−0·1

The mean for the whole period 1847–1901 is 0″·0. It does not seem to me that these figures give any evidence—though their weight is small—of a systematic difference between the observations with the old transit circle from 1847–1851 and those with the present instrument.

The negative values for periods 94–98 suggest some systematic error of the transit-circle observations during this period, but viewed with the whole series of periods to 1901 it is quite possible that the agreement of sign for these years is accidental.

8. The figures which Professor Turner gives (Table V., p. 411) show that a very liberal allowance must be made for accidental error. The figures he gives are as follows :

Day.	Old.	New.	Day.	Old.	New.
	″	″		″	″
16	0·0	+0·1	21	−0·7	+0·6
17	−1·1	−0·1	22	−0·2	+1·0
18	+1·1	0·0	23	+0·7	+0·8
19	−0·2	+0·4	24	+0·1	+0·8
20	−1·1	+0·2	25	−0·5	+1·2

The figures in the column "Old" seem wholly accidental, although except for day 25 each of these figures is derived from the differences of from 25 to 50 observations made with the two instruments. The systematic character of the figures in the column "New," as far as it is caused by the transit circle, seems to agree with the error noted in § 7 as existing in Mr. Cowell's periods 94–98, and in that case would not occur in a comparison extending over a longer period.

9. In conclusion, although the value Mr. Cowell gave for the solar parallax seems to me to be true within the limits he assigned, I do not think any discussion of the errors of the observations will materially narrow the limits. Observations of the Moon, whether meridian, altazimuth, or occultations, will always have the "limb" difficulty.

The agreement between the value he finds and the value obtained in other ways seems to me to show that in a long series of observations the errors largely balance one another, and that as the parallactic inequality and the variation are the terms which are affected systematically, the accuracy of the determination of short-period terms not so affected will be very great.

Comments on Mr. Dyson's "Note on the Instrumental Errors affecting Observations of the Moon." By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

(*a*) Mr. Dyson courteously submitted the preceding paper to me in MS., and I offer the following comments upon it which it may be convenient to have in the same number as the paper itself.

(*b*) Paragraphs 1, 2, 3 of his note call for no remark from me.

(*c*) In § 4 our views begin to diverge, but only in the phrase "to obtain from the accordance of the separate determinations a measure of their accuracy." The danger of this test is well known even when all conceivable systematic errors have been removed. When there is an obvious source of systematic error still affecting the observations, I doubt whether the test can be applied. But we shall come across this point more definitely later.

(*d*) On § 5 I would urge that if there really are "no means available for determining the instrumental correction, &c., &c." —that is, if the solar parallax as determined from observations of the Moon is affected with an entirely unknown systematic error, then it is a pity to publish the result. To do so is quite likely to make trouble, from the value being carelessly used irrespective of the limitation on some occasion or another; and that it is so near the probable truth only increases this likelihood. And what good is it? We know that the solar parallax is there or thereabouts; what we want to know now is, if possible, where *exactly* it is.

(*e*) On § 6 I have no comment.

On § 7 I have not yet before me the paper of Mr. Cowell's quoted, but I gather that the column referred to only gives the values of sin D. This only gives information concerning a very small part of the instrumental error for two reasons.

Firstly, if the instrumental error be expressed as a series

$$a_1 \sin D + b_1 \cos D + a_2 \sin 2D + b_2 \cos 2D + \text{ &c.}$$

the coefficient b_1 at any rate is likely to be large. For the instrumental error is almost certain to be quite different at new moon and full moon, *i.e.* at $D = 0°$ and $180°$. Now *sin D* has the same value at these points, and thus terms like $a_1 \sin D$ alone cannot represent errors of this kind. To trace changes in instrumental errors we must have at any rate the coefficients of cos D as well as those of sin D, and probably the higher coefficients also.

Secondly, *for expressing instrumental error* I do not trust the coefficients found in this way at all, for the reason that we have so few observations near new moon. It must be remembered that the case is totally different from that of the determi-

nation of a function whose form is known, in which case we can find its coefficient from a single value even. Suppose, for instance, we had two observed functions f_1 and f_2 thus:

D =	0°	30°	60°	90°	120°	150°	180°
f_1 =	0	1	2	3	(2)	(1)	(0)
f_2 =	0	1	2	3	(4)	(5)	(6)

the first rather like 3 sin D and the second rather like 3(1 − cos D); and suppose that we had no observations beyond D = 90°, so that we were in entire ignorance of the values except from 0° to 90°, where the functions coincide. Whatever analytical process we employed to represent them by series of sin D and cos D &c., we should of course get the same result for f_1 as for f_2; and yet the accordance would be quite illusory. This is only exaggerating for illustration what Mr. Cowell has done, if I understand him rightly. Observations near full moon where the instrumental errors are probably nearly constant (because the conditions remain nearly the same) receive most weight and largely control the analytical results. Thus, to adapt the above illustration a little nearer to the facts, suppose we had two quite different instrumental errors

D =	0°	30°	60°	90°	120°	150°	180°
f_1 =	3	3	2	1	0	(−1)	(−2)
f_2 =	3	3	4	5	6	(7)	(8)
Weight =	10	10	6	2	1	0	0

The great weight (of course purposely exaggerated) assigned to the observations near 0° would give nearly similar coefficients for the periodic terms sin D and cos D &c. and thus entirely mask the real and serious instrumental difference.

(*f*) With § 8 I entirely agree. Perhaps the large accidental errors of the altazimuth observations were scarcely sufficiently emphasised in my paper.

(*g*) With regard to § 9, as also to § 4 preceding, it seems to me very important to exclude any idea that the length of the series of observations discussed can improve the value of the solar parallax thus obtained. I do not see how a possible systematic error can be in any way diminished by multiplying observations, unless it is equally likely to occur with positive and negative signs; and in the case before us this is unlikely. At any rate the *onus probandi* is on those who make this assumption. If it is thoroughly understood that the value of the solar parallax is put forward, not as an exact value, but as one affected with an unknown systematic error, my objection is so far met, though I think it a pity to multiply imperfect values.

(*h*) But I will say in conclusion that I do not feel sure that Mr. Dyson and Mr. Cowell are right to despair of determining

this error, or at least of narrowing its limits. The altazimuth observations could be cleared of the error for high and low moon, with some trouble perhaps, but still it could be done, since observations in the same phase but at different altitudes could be compared. And then they could be used to throw some light on the errors of the meridian instruments. This would be one way. Another way would be to try the effect of restricting the determination of sin D to the days near full moon, where the instrumental or observational errors may be supposed with more legitimacy to remain constant, and where sin D is nevertheless varying rapidly. This would be easy to do, and a comparison of the results with those previously obtained from the whole material would perhaps tell us something valuable about the limits of error.

P.S.—After the above had been sent to the printers the following note was received from Professor Newcomb. In an accompanying letter he leaves open the question of printing it; but as it contains at least one important correction to what has already been said, there can be no doubt of the propriety of publishing it here. I refer to the question of the *division errors* of the azimuth circle of the altazimuth, to which Professor Newcomb draws attention. For my own part I was quite under the impression that these had been applied, so that Professor Newcomb's remark would have lost point; but on submitting this opinion to Mr. Dyson he replied (1904 May 20):

> "No division errors were applied before 1880, when the circle was redivided. This I have verified from the computing books. A determination was made in 1847, but apparently never used. I don't know how good it is."

Hence some preliminary work, which I had overlooked, is apparently necessary before the altazimuth observations can be used to discuss systematic errors.

On another point a word may be added to Professor Newcomb's note. The observations of altitude are not *all* made on the lower limb; in the second and third quarters the upper limb was sometimes observed near rising and setting respectively, as may be seen in the printed volumes. Whether there is enough material to determine the error of diameter from these I can scarcely say. Further, as I have remarked in a footnote on p. 405 of this volume, the combination of the altitude observations with those of azimuth may reduce the effect on the longitude in all cases to a simple error of semi-diameter.

Finally, I am glad to have confirmed by such an authority the view expressed in the last sentences of my remarks above—that in Professor Newcomb's words "the best result of meridian observations is to be obtained by throwing out all the observations made in daylight and eliminating the correction to the semi-diameter from the equations themselves." This experiment would in any case be most interesting.

Remarks on the Determination of the Parallactic Inequality of the Moon. By Simon Newcomb.

The following remarks are suggested by the papers of Professor Turner and Mr. Cowell in the December and March numbers of the *Monthly Notices* (vol. lxiv., Nos. 2 and 5). They relate to sources of possible systematic differences between the results of meridian and altazimuth observations of the Moon and possible systematic errors in a determination of the parallactic inequality which are not fully considered in the papers referred to.

Beginning with the meridian observations, it is necessary, in order to have a determination of the parallactic inequality free from systematic error, that all the observations used should be free from such systematic errors as are a function of the age of the Moon. The practical semi-diameter of the Moon, as it enters into the meridian observations, should not be regarded as a constant. Saying nothing of personal equation, which we assume to be eliminated from the mean during any half-lunation, the practical semi-diameter of the Moon is the true semi-diameter augmented by radiation. The latter is a function of the contrast between the brightness of the Moon's limb and that of the sky. It is therefore greater after the end of twilight and least when the Sun is above the horizon. It will also be less at the full moon on account of the greater obliquity at which the rays strike, and from the fact that the illumination of the extreme limb will be deficient, owing to many points on it being thrown into shadow when the observer is not practically in line with the Sun. For this reason we must expect the semi-diameter determined from transits to be too small. Under these circumstances it seems to me the best result of meridian observations is to be obtained by throwing out all observations made during daylight, and eliminating the correction to the semi-diameter from the equations themselves. I am inclined to think that we should get the best results if the discussions were confined to observations between November 1 and February 15. Each observation would give an equation of condition between the corrections to the semi-diameter and to the parallactic inequality.

The altazimuth observations are liable to errors, which are a function of the azimuth, unless the errors of graduation are determined and the instrument is so constructed as to be perfectly free from wabble. Observations with this instrument near and before the time of first quarter will commonly be made in the south-west or west, while those near the third quarter will be made in the south-east or east. We must also recognise the possibility of systematic errors in altitude as well as those of semi-diameter. A little consideration will show the following results of these errors :—

1. A constant error in altitude will have opposite effects

on the Moon's longitude at first and third quarters. It will therefore enter with its full effect into the parallactic inequality, but will be eliminated from the mean longitude.

2. Since all the observations of altitude must be made on the Moon's lower limb, the effect of error of semi-diameter will be the same as that of the altitude.

3. The errors in azimuth will practically have the same effect as those in right ascension, both as regards the limb and the errors which are a function of the azimuth itself.

For these reasons we must not expect the two methods to agree. Until we free the altazimuth observations from the source of error in question, it seems to me not safe to rely upon them for the parallactic inequality. To do this requires a more careful study of the instrument and the method of using it than I am aware of having been made.

Washington: 1904 *May* 11.

Methods of Correcting Moon's Tabular Longitude.

By P. H. Cowell.

In previous papers I have given a list of about a dozen corrections applied to Hansen's tabular longitudes from 1847·0 onwards. I now give a list of corrections to Airy's tabular longitudes, 1750–1851, that have already been entered, though the additions have not as yet been completed.

Terms whose arguments are multiples of the same angle are combined in one table, and in the following list are given under a single reference number; for example, the first table applies the correction

$$-2''{\cdot}58 \sin D - 0''{\cdot}20 \sin 2D - 0''{\cdot}37 \sin 3D - 0''{\cdot}46 \sin 4D$$

all at once.

The only exception to this is the terms in g' and $2g'$, where the secular variation of the former made it necessary to treat the latter separately.

The first list contains terms of short period; the second list terms of longer period:

Ref. No.	Argument.	Coefficient of Correction Applied.	Coefficient used by Airy *plus* Coefficient of Correction.	Remarks.
		"	"	
1	D	−2·58	+ 124·68	Parallactic Inequality
1	2D	−0·20	− 2370·10	Variation
1	3D	−0·37	+ 0·53	
1	4D	−0·46	+ 13·94	

Ref. No.	Argument.	Coefficient of Correction Applied.	Coefficient used by Airy *plus* Coefficient of Correction.	Remarks.
		"	"	
2	$-2g-g'$	+0·38	+ 7·68	
3	$-g-g'$	−1·20	+ 109·90	
4	$g-2g'$	+0·49	+ 2·59	
5	$g+2\omega-2\omega'$	−0·45	− 2·45	
6	$2g-g'+2\omega-2\omega'$	−0·88	... 24·48	
7	$-g-2g'+2\omega-2\omega'$	+0·44	+ 13·24	
8	$g-3g'+2\omega-2\omega'$	−3·30	+ 206·40	
9	$2g-3g'+2\omega-2\omega'$	−0·38	+ 165·52	
10	$3g-3g'+2\omega-2\omega'$	+0·63	+ 14·63	
11	$4g-3g'+2\omega-2\omega'$	+0·56	+ 1·16	
12	$3g-4g'+2\omega-2\omega'$	+0·44	+ 0·74	
13	$2g+g'+2\omega$	+0·42	+ 0·42	Term omitted by Airy.
14	$g+2\omega$	−2·36	− 39·56	
15	$2g+2\omega$	−0·62	− 411·62	
16	$-g+3g'+2\omega'$	+0·38	+ 0·38	Term omitted by Airy.
17	$2g+2g'+2\omega'$	+0·35	+ 0·55	
18	$g+\omega-\omega'$	+0·70	+ 17·90	
19	$-g-g'+\omega-\omega'$	−0·78	− 1·58	
20	$g-3g'+3\omega-3\omega'$	−0·61	− 1·21	
21	$g-4g'+4\omega-4\omega'$	+0·60	+ 1·10	
22	$3g-4g'+4\omega-4\omega'$	+0·46	+ 38·46	
23	$5g-4g'+4\omega-4\omega'$	+1·03	+ 1·93	
24	$2g-5g'+4\omega-4\omega'$	+1·55	+ 2·75	
25	$3g-5g'+4\omega-4\omega'$	+0·58	+ 4·38	
26	$4g-5g'+4\omega-4\omega'$	+0·58	+ 1·78	
27	$2g-2g'+4\omega-2\omega'$	+0·36	− 0·54	
28	$4g-2g'+4\omega-2\omega'$	−2·34	− 5·74	
29	$5g-2g'+4\omega-2\omega'$	−0·40	− 1·00	
30	$3g-3g'+4\omega-2\omega'$	−0·43	− 0·43	Term omitted by Airy.
31	$4g-6g'+6\omega-6\omega'$	+0·54	+ 0·54	„ „
32	$2g-5g'+2\omega-4\omega'$	−0·40	0·00	Airy's wrong term.
33	$g+2\varpi-2\mathrm{J}$	−0·893	− 0·893	Jupiter term.
34	$g+2\varpi-3^{\mathrm{J}}+7^{\circ}$	−0·316	+ 0·316	„ „
35	g	−2·10	+ 22639·50	Anomaly.
35	$2g$	−0·46	+ 769·04	
35	$3g$	−0·58	+ 36·12	
36	$2\mathrm{D}-g$	+0·738	+ 4586·338	Evection.
36	$4\mathrm{D}-2g$	−3·75	+ 30·75	

Ref. No.	Argument.	Coefficient of Correction Applied.	Coefficient used by Airy *plus* Coefficient of Correction.	Remarks.
I.	$-g'+2\omega-2\omega'$	$+3''.80$	$+2''.40$	
II.	$-g'+\omega-\omega'$	-0.60	-18.60	
II.	$-2g'+2\omega-2\omega'$	-0.65	$+211.75$	
III.	$-3g'+2\omega-2\omega'$	$+0.86$	$+8.66$	
IV.	$-g'$	$+1.03$ -1.63 T	$+669.63$ -1.63 T	Annual equation. T in centuries from 1800
V.	$-2g'$	-0.40	$+7.50$	
VI.	$+2g'+2\omega'$	-0.30	-55.20	
VII.	V − E	$+0.30$	-0.80	Venus term
VIII.	$2V-3E+85°$	-0.30	-0.30	„ „

Subsequently a term in cos g will probably be added to the above list; but I propose carrying out analyses for g and D without applying any further short period corrections.

I now proceed to explain the method employed.

The transits of the Moon, whether observed or not, are numbered consecutively, counting from 1750 September 13^d 10^h as No. 1; for convenience they are then divided into forties; and the 29th and 30th batch of forty are shown in the columns headed [29] and [30] respectively. The blank in the first line of column [29] informs us that the Moon was not observed at the corresponding transit which is clearly 28×40 lunar days later than column [1], line 1, which corresponds to 1750 September 13^d 10^h. If anyone is interested in expressing this date in terms of the civil calendar, it is easy to do so, by taking the lunar day as 1·03505 of a solar day. The entry −50 in the second line informs us that Airy's tabular longitude of the Moon exceeded the observed longitude at the corresponding transit by −50 units of one-tenth of a second of arc. This is the result of pure copying from Airy's Greenwich Lunar Reductions, with the following modifications only: (i) the second decimal of a second is omitted; (ii) the decimal point is omitted; (iii) the sign is changed, as Airy gives observed *minus* tabular; (iv) means are taken when Airy gives more than a single result, that is to say, when both first and second limbs have been observed. The third line shows that the corresponding transit was not observed; then follow three observations, and then seven unobserved transits (during which it is clear that new moon must have fallen) and so on.

TABLE 7.

	7	[29]	7	[30]	1	2
1	...	...	...	...	A + 1	− 4
2	− 1	− 50	...	...	+ 2	− 4
3	=	...	−	−	+ 3	− 4
4	+ 1	− 20	− 4	− 72	+ 4	− 3
5	+ 2	− 46	− 4	− 74	+ 4	− 2
6	+ 3	− 38	...	...	+ 4	− 1
7	...	...	− 3	− 28	+ 4	0
8	...	...	− 2	− 12	+ 4	+ 1
9	...	...	...	...	+ 3	+ 3
10	...	...	0	+ 63	+ 2	+ 3
11	...	...	+ 1	+ 78	+ 1	+ 4
12	...	...	+ 2	+ 71	− 1	+ 4
13	...	...	+ 3	− 2	− 2	+ 4
14	+ 1	− 12	+ 4	− 27	− 3	+ 4
15	0	− 29	...	...	− 4	+ 4
16	...	...	+ 4	− 31	− 4	+ 3
17	− 2	− 45	...	...	− 4	+ 2
18	− 3	+ 19	...	...	− 4	+ 1
19	...	...	...	...	− 4	− 1
20	- -	...	...	...	- 3	− 2
21	− 4	− 13	...	...	− 3	− 3
22	− 4	+ 64	0	0	− 1	− 4
23	− 4	+ 87	− 1	+ 31	0	− 4
24	− 3	+ 70	...	...	B + 1	− 4
25	...	...	...	...	+ 2	− 4
26	...	...	...	...	+ 3	− 4
27	...	...	...	...	+ 4	− 3
28	...	...	...	...	+ 4	− 2
29	...	...	...	...	+ 4	− 1
30	...	...	...	...	+ 4	0
31	...	...	− 3	− 41	+ 4	
32	+ 4	− 63	...	...	+ 3	
33	...	...	...	...	+ 2	
34	...	...	0	- 83	+ 1	
35	...	...	+ 1	- 89	0	
36	...	...	...	...	− 1	
37	...	...	...	...	− 2	
38	...	...	+ 4	− 21	− 3	
39	...	...	+ 4	+ 26	− 4	
40	...	...	+ 4	+ 46	− 4	

Precepts

	c	l
B	29	4
B	30	11
B	31	18
B	32	25
A	33	32
B	34	39

The computing form annexed is to be considered as consisting of three, or rather four, separate pieces of paper. The first piece contains column [29] as its extreme right-hand column, to the left lies the various corrections of which correction 7 is alone shown; the second piece shows column [30] and correction 7 in a similar manner; the third and fourth pieces are mere strips which may be referred to as table 7, columns 1 and 2, respectively.

The following is the process of entering the corrections: the last three entries of table 7, column 2, are held in juxtaposition to the first three lines of column [29]; that this is the right position is inferred from the fact that the preceding entry of table 7 has been previously found to apply to column [28] since 40. Holding the slip in his left hand and to the left of column [29], the computer then enters the correction whenever the transit has been observed, *e.g.* he does not copy −2 and 0 in the first and third lines, but he does copy −1 in the second line. Having now come to the end of table 7 he draws a double line as shown. When table 7 comes to an end it is always proper to begin again, either at the beginning (the first entry is marked A) or at the 24th entry, which is marked B. To ascertain at which of these two points to begin in the present instance, it is necessary to refer to the Precepts, an extract from which is annexed, stating that at column [29], line 4, the proper entry is B. This precept serves the further purpose of preventing mistakes, for, supposing for example that in turning over from column [28] to column [29] the computer slipped one line, he would discover the mistake on finding that the precept did not appear to come at the right line; he would then go back to the preceding precept, which has presumably come right, and make the necessary alterations, and the mistake is prevented from continuing undiscovered throughout an indefinite number of columns.

The plan of the table is obvious. The entries are the products of −4·4 by the sines of angles in an arithmetical progression, the common difference being 15°·322690, the movement of the argument in one lunar day. The corrections thus applied correspond to mean lunar noons, but no serious error is committed by applying them at apparent lunar noons.

To save a little labour in constructing the table the argument of the middle entry (in the present case the 35th) is taken as exactly zero. The second half of the table is then constructed, and the first half follows from it by a change of sign and reversing the order of the entries.

I give below the full calculation for the table:

Entry.	Argument.	Entry.	Argument.
35	0°·00	58	0° − 7°·35
46	180° − 11°·34	59	0° + 7°·98
47	180° + 3°·99	70	180° − 3°·36

n	$\log \left\{\frac{2n-1}{8\cdot8}\right\}$	Anti-log. sine.	Residual after subtracting multiples of 15°·332690.
		°	°
1	0555	6·52	6·52
2	5326	19·93	4·60
3	7545	34·63	3 96
4	9006	52·70	6·70
		90·00	13·33

The above little computation is sufficiently explained by its headings; the last column informs us (second line for example) that $4\cdot4 \times \sin(15°\cdot332690+\theta) = 1$ if θ be less than 4·60 and $= 2$ if θ be greater than 4·60; the last line informs us that $5 \times 15°\cdot332690+\theta$ exceeds 90° if θ exceeds 13°·33. Table 7 is now constructed six entries at a time by merely referring to the last column of the above computation; for instance, the entries Nos. 46–41 in the reverse order are seen to be −1, −2, −3, −4, −4, −4, by merely comparing 11°·34 (see value of argument for entry No. 46) with the angles that stand in the above column.

I come now to the formation of the precepts. The movement of the argument in seventy lunar days is $3 \times 360° - 6°\cdot7117$; in forty-seven lunar days is $2 \times 360° + 0°\cdot63643$. From this it may be seen that to "Return to A" adds $-6°\cdot7117$ to the excess of the true argument (*i.e.* the true value of the argument at the nearest mean lunar noon) over the argument of the table; while to "Return to B" adds $+0°\cdot63643$ to the same excess. Consequently it follows that the difference between the true argument and the argument of table can always be kept less $\frac{1}{2}\{6°\cdot7117 + 0°\cdot63643\}$ or 3°·67, and the maximum error committed will be less than 0″·44 × circular measure of 3°·67, or less than 0″·03. Had it been considered desirable the error might have been confined to narrower limits by using a larger table. The numbers 47 and 70 are readily found by turning $\frac{15°\cdot332690}{360°}$ into a continued fraction, the first two convergents are $\frac{1}{2}\frac{1}{3}$ and $\frac{2}{4}\frac{3}{7}$; of the wo numbers 47 and 70, therefore, the former is the denominator of the last convergent, and the latter the sum of the denominators of the two last convergents.

It will be seen that the method is perfectly general; it can be applied to corrections of any size and to any degree of accuracy at the cost of enlarging the table. If the coefficient of the correction is N times the maximum permissible error, the table must consist of $2\pi N$ entries at least. If, however, the table had been required for an ephemeris instead of for a series of discontinuous dates, it would be reasonable to throw such operations as changing signs and copying backwards upon the computer, and the size of the table could be divided by 4, making $\frac{\pi}{2}N$ entries at the least. The use of the table involves mere copying without interpolation, and it is clear that any table professing to tabulate a quantity

that may take all values from 1 to N without interpolation correctly to a single unit, must contain N entries. The coefficient of efficiency of the method may therefore be taken as $\frac{2}{\pi}$ (perfection being taken as unity).

This slight loss of efficiency is clearly not a high price to pay for reducing the entering to pure copying without interpolation, and abolishing the calculation of the arguments for the individual observations.

As I pointed out in the *Observatory* for 1903 July, in the case of large corrections interpolations may be grafted on. Suppose, for instance, it were required to tabulate $22639''\cdot50 \sin g$ correctly to $0''\cdot01$, the elliptic inequality for every ten minutes of mean solar time. A table of 1000 entries would give this quantity for equal increments of the argument corresponding to the movement in ten minutes from 0° to 90°. Pure copying of this table (backwards or forwards, with or without change of sign) would give all the quantities calculated with arguments wrong by the same constant Δg, say. It would then remain to calculate $22639''\cdot50\ \Delta g \cos g$, an operation not outside the limits of Crelle or Cotsworth's multiplication tables, if the correction be tabulated for the maximum value of Δg. The term in Δg^2 is not required. The modification is obvious, if the interval of the ephemeris is not to be 10 minutes, but say 12 hours; the table would then have to extend over 72 quadrants instead of one, and the number of entries in each quadrant will be one seventy-second part of the number (1,000) contemplated in the previous case.

There remains a special case to be alluded to, which is best illustrated by considering correction No. 23. The movement of the argument in one lunar day in this case is $63°\cdot995060$, and the successive convergents to $\frac{63°\cdot995060}{360°}$ are $\frac{1}{5}, \frac{1}{6}, \frac{2}{11}, \frac{3}{17}, \frac{8}{45}, \frac{283}{1592}$.

To form a table with $1592+45$ entries would be a labour altogether out of proportion to the end in view. A table, however, of $45+17$ entries is rather too small, as it would permit errors of $0''\cdot07$. Four tables of 45 entries each were therefore formed. The tables are lettered A, B, C, D. In each table the arguments are in arithmetical progression with a common difference of $63°\cdot995060$. The following extract from the value of the arguments

$D\ 1 = -a$	$A\ 1 = 180° - D45$	$B\ 1 = 180° + D\ 1$	$C\ 1 = 180° + A\ 1$
$D32 = 180° + 3°\cdot85 - a$	$A14 = 180° - D32$	$B32 = 180° + D32$	$C14 = 180° + A14$
$D45 = -0°\cdot22 - a - 63°\cdot995$	$A45 = 180° - D\ 1$	$B45 = 180° + D45$	$C45 = 180° + A45$

shows that when one table has been constructed, the remaining tables follow by a change of sign, or a reversal of order, or both. It remains to explain the principles that determine the value of a, and the nature of the precepts.

When the computer reaches the end of any table, the precepts

usually direct him to recommence the same table. This, however, involves increasing the excess of the true argument over the argument of the table by $-0°{\cdot}22$, and cannot be indefinitely repeated.

Occasionally, therefore, the precepts direct him to substitute

A14 for D1	C14 for B1
B2 for A1	D2 for C1

In other words, from time to time he has to change his table, always following the order of the alphabet, and having changed to go round and round the same table till the time comes for the next change.

Now, as the precepts are written, before settling the value of a it will be seen that the excess of the arguments D1, B1 over A14, C14 respectively is $3°{\cdot}85-2a$; and the excess of the arguments A1, C1 over B2, D2 respectively is $2a$. Hence we are constantly increasing the excess of the true argument over the argument of the table by $-0°{\cdot}22$, and we have the power, whenever we choose, of affecting this excess by a further increment, $3{\cdot}85-2a$ or $2a$. It is now clear that we ought to take $a=0°{\cdot}96$ so as to make them two quantities each equal to $1°{\cdot}92$, when it will be seen that the power of adding $+1°{\cdot}92$ at will enables us to keep the difference between the true argument and the tabular argument below the numerical value $0°{\cdot}96$, corresponding to a maximum error $1''{\cdot}03\times0°{\cdot}96-0''{\cdot}02$ in the use of the table.

Moreover, the method of tabulating the errors is convenient for the analysis of errors as well as applying corrections. Suppose, for example, it is required to analyse the errors for possible terms $g+g'$, or $g+g'$ with the addition of any long-period argument such as $\omega-\omega'$. We may take as an auxiliary angle θ, whose movement in one lunar day is $+14°{\cdot}4$ and whose value is zero at each twenty-fifth day of each period of analysis. The ten sheets containing the 10×40 lunar days that go to one period of analysis are taken, and a cross is placed opposite columns 1 and 6, lines 25; columns 3 and 8, lines 20; columns 5 and 10, lines 15; columns 7 and 2, lines 10; columns 9 and 4, lines 5. The sheets are then placed so that the crosses fall in a row, whereupon the value of θ for all the observations in any row is the same. The sums are then taken down upon a convenient form for analysis into sine and cosine, and checked by the condition that the sum of the sums must be equal to the sum of all the errors for the whole period of analysis, a total that has been previously formed.

Now the movement of $g+g'-\omega+\omega'$ exceeds the movement of the auxiliary angle by $11°{\cdot}1221$ in a period of analysis. Hence an error with argument $g+g'-\omega+\omega'$ will discover itself by a periodicity in a little less than 33 periods of analysis in the coefficients of $\sin\theta$ and $\cos\theta$.

Further Analyses of Moon's Errors with Mean Elongation as argument, 1847–1901. By P. H. Cowell.

The present analyses are based upon the errors after thirteen corrections have been applied to Hansen's tabular places. These corrections are given in previous papers, where the periods of analysis are also defined.

In the annexed table the first column gives the number of the period of analysis. The next two columns are copied from last month's paper. The next three columns are fresh matter, and they are sufficiently distinguished by their headings. The meaning of the word "apparent" is that these are the corrections that are found by analysis upon the successive assumptions that each one alone exists. The subsequent columns, headed "resolved values," take account of the co-existence of the various separate errors. The relation between the "apparent" and "resolved" values is clearly dependent upon the distribution of the observations relatively to the age of the Moon. Relations were investigated, based upon the whole of the 48 periods of analysis (1847–1901). These relations should not strictly be applied to the individual periods, for any particular period may have an abnormal distribution of observations, but the error due to this cause has been allowed to coalesce with the accidental errors in the belief that they will not be so distributed as to sensibly affect the coefficients of deduced periodic corrections.

I now investigate the formulæ used. The notation is μ, δ_1, δ_2, Δ_1, Δ_2 stand respectively for the correction to semi-diameter, coefficients of sin D, sin 2D, cos D, cos 2D (resolved values). Accented letters denote apparent values. It will be seen that μ, δ_1, δ_2 fall into one group, Δ_1, Δ_2 into a second group, and that the two groups are kept separate.

First as to μ, δ_1, δ_2; in the December *Monthly Notices* I obtained from 5,647 observations normal equations:

$$5647\mu + 3788\delta_1 - 2327\delta_2 = \Sigma\pm\epsilon,$$
$$3788\mu + 3074\delta_1 - 1366\delta_2 = \Sigma\epsilon \,.\, \sin \mathrm{D},$$
$$-2327\mu - 1366\delta_1 + 2645\delta_2 = \Sigma\epsilon \,.\, \sin 2\mathrm{D},$$

ϵ being the error of any observation and the 48 times 400 lunar days being taken together.

For each of the present periods of analysis I put, n denoting the number of observations.

$$\mu' = \frac{1}{n}\Sigma\pm\epsilon$$

$$\delta'_1 = \frac{2}{n}\Sigma\epsilon \sin \mathrm{D}.$$

$$\delta'_2 = \frac{5647}{2645n}\Sigma\epsilon \sin 2\mathrm{D}.$$

Analyses of Tabular minus *Observed Longitudes of Moon for Periods 86–133 in Tenths of a Second of Arc.*

	Apparent Values of					Resolved Values of				
Period.	Semi-Diameter Term.	Co-efficient of sin D.	Co-efficient of sin 2D.	Co-efficient of cos D.	Co-efficient of cos 2D.	Semi-Diameter Term.	Co-efficient of sin D.	Co-efficient of sin 2D.	Co-efficient of cos D.	Co-efficient of cos 2D.
86	+ 1	+ 2	0	− 5	+ 2	− 1	+ 3	+ 1	− 8	− 2
87	− 8	−11	+ 1	− 2	+ 1	−13	+ 2	−10	− 2	0
88	− 2	− 1	+ 3	−10	+ 4	− 7	+ 9	+ 1	−16	− 4
89	0	0	0	− 6	+ 4	0	0	0	− 3	+ 3
90	− 4	− 5	+ 3	− 7	+ 4	− 6	+ 2	− 1	− 6	+ 1
91	+ 3	+ 5	− 2	0	+ 2	0	+ 5	0	+ 8	+ 6
92	− 5	− 8	+ 6	− 3	+ 2	+ 2	− 8	+ 3	− 2	+ 1
93	− 2	0	+ 5	− 5	+ 4	− 9	+13	+ 4	0	+ 5
94	− 3	− 5	0	+ 1	− 1	− 2	− 4	− 4	− 1	− 1
95	− 6	− 9	+ 4	+ 7	− 4	− 4	− 4	− 1	+ 6	− 1
96	− 8	−12	+ 5	− 1	− 4	− 5	− 6	− 3	−19	−15
97	−11	−13	+11	+ 5	− 2	−16	+ 9	+ 1	+ 8	+ 2
98	− 6	− 9	+ 5	+ 1	+ 2	− 2	− 5	0	+11	+ 8
99	0	− 1	+ 1	− 4	+ 2	+ 5	− 6	+ 2	− 5	− 1
100	+ 3	+ 4	− 5	− 3	+ 2	+ 1	+ 1	− 4	− 2	+ 1
101	0	− 1	0	+ 1	− 1	+ 4	− 6	+ 1	− 1	− 1
102	+ 3	+ 4	− 5	+10	− 7	+ 1	+ 1	− 4	+ 3	− 5
103	+ 3	+ 4	− 4	+ 2	+ 1	+ 2	+ 1	− 2	+10	+ 6
104	− 3	− 6	+ 4	0	+ 1	+ 6	−12	+ 3	+ 4	+ 3
105	+ 1	+ 1	+ 2	− 4	+ 2	+ 5	− 4	+ 5	− 5	− 1
106	+ 1	+ 3	0	+ 3	− 1	− 5	+ 9	0	+ 6	+ 2
107	+ 1	+ 2	0	+ 5	− 2	− 1	+ 3	+ 1	+ 8	+ 2
108	+ 4	+ 6	0	+ 3	− 1	+ 5	+ 1	+ 5	+ 6	+ 2
109	+ 8	+13	− 4	0	0	+ 2	+10	+ 3	0	0
110	+ 3	+ 2	− 3	− 1	0	+11	−11	+ 1	− 3	− 2
111	− 2	− 3	0	− 3	+ 2	− 3	− 1	− 3	− 2	+ 1
112	0	− 1	0	− 5	+ 4	+ 4	− 6	+ 1	0	+ 5
113	+ 1	+ 2	− 3	+ 2	+ 1	− 4	+ 5	− 4	+10	+ 6
114	+ 2	+ 3	− 4	−12	+ 6	− 2	+ 3	− 4	−13	− 1
115	+ 3	+ 3	0	− 9	+ 6	+10	− 7	+ 5	− 4	+ 4
116	+ 3	+ 3	− 4	−14	+ 6	+ 6	− 5	− 2	−22	− 5
117	+ 4	+ 4	− 5	− 5	+ 3	+ 8	− 7	− 1	− 4	+ 2
118	+ 3	+ 4	− 2	− 2	+ 1	+ 4	0	+ 1	− 2	0
119	+ 1	+ 2	− 4	+ 5	− 3	− 5	+ 5	− 6	+ 4	− 2
120	0	0	− 3	− 4	0	− 3	+ 2	− 5	−13	− 7
121	+ 3	+ 5	0	0	+ 1	+ 2	+ 4	+ 4	+ 4	+ 3
122	+ 8	+12	− 5	− 3	+ 1	+ 5	+ 6	+ 3	− 6	− 2
123	+ 3	+ 5	− 3	− 2	+ 1	− 1	+ 6	− 1	− 2	0
124	+ 3	+ 5	− 2	− 6	+ 4	0	+ 5	0	− 3	+ 3
125	+ 1	+ 1	− 6	− 6	+ 4	− 3	+ 1	− 8	− 3	+ 3
126	− 4	− 6	+ 2	+ 1	− 1	− 3	− 3	− 2	− 1	− 1
127	+ 2	+ 2	− 2	− 4	+ 4	+ 4	− 4	0	+ 3	+ 6
128	0	− 1	0	0	− 1	+ 4	− 6	+ 1	− 4	− 3
129	− 4	− 4	+ 4	− 4	+ 2	− 9	+ 7	0	− 5	− 1
130	− 3	− 5	+ 2	− 5	+ 1	0	− 5	0	−12	− 5
131	− 5	− 6	+ 4	− 4	+ 2	− 8	+ 4	− 1	− 5	− 1
132	− 1	− 2	0	− 3	+ 2	+ 1	− 3	− 1	− 2	+ 1
133	− 1	− 2	+ 3	+ 4	+ 1	+ 4	− 5	+ 4	+17	+10

Subsequently I deduce μ, δ_1 and δ_2 from the formulæ

$$\begin{aligned} 5647\mu + 3788\delta_1 - 2327\delta_2 &= 5647\mu' \\ 3788\mu + 3074\delta_1 - 1366\delta_2 &= \tfrac{1}{2}5647\delta_1' \\ -1366\mu + 2327\delta_1 + 2645\delta_2 &= 2645\delta_2' \end{aligned}$$

It will be seen that μ', δ_1', δ_2' are purely arbitrary auxiliary quantities. The treatment is unfortunately not quite symmetrical. I wish that I had taken for δ_1', $\frac{1}{2} \cdot \frac{5647}{3074}$ times the value which I have taken. The explanation is that when I came to Δ_1, Δ_2 I was forced into definitions corresponding to that of δ_2' ; I was in time to define δ_2' accordingly, but too late for δ_1'. Of course nothing is lost but symmetry.

Solving the equations for μ, δ_1, δ_2, in terms of μ', δ'_1, δ'_2, I obtain

$$\begin{aligned} \mu &= \quad 7{\cdot}285\mu' - 3{\cdot}980\delta'_1 + 1{\cdot}079\delta'_2 \\ \delta_1 &= -7{\cdot}960\mu' + 5{\cdot}537\delta'_1 - 0{\cdot}597\delta'_2 \\ \delta_2 &= +2{\cdot}306\mu' - 0{\cdot}637\delta'_1 + 1{\cdot}646\delta'_2 \end{aligned}$$

and with the help of these equations the resolved values have been obtained from the apparent values.

It follows that

$$n\mu = \Sigma\varepsilon\{\pm 7{\cdot}285 - 7{\cdot}960 \sin D + 2{\cdot}306 \sin 2D\}$$

$$\frac{n}{2} \cdot \delta_1 = \Sigma\varepsilon\{\mp 3{\cdot}980 + 5{\cdot}537 \sin D - 0{\cdot}637 \sin 2D\}$$

$$\frac{n}{2}\delta_2 = \Sigma\varepsilon\{\pm 1{\cdot}153 - 0{\cdot}637 \sin D + 1{\cdot}756 \sin 2D\}$$

Throughout any period of analysis the value of D is supposed to increase by $720° \div 57$ between one transit of the Moon and the next, and the small error of this assumption is prevented from accumulating by introducing the necessary discontinuity in passing from one period of analysis to the next. Only 57 values of D are therefore recognised—namely, multiples of $720° \div 57$; and the above factors merely change sign as we pass from the multiple by p to the multiple by $57 - p$.

I have therefore tabulated for values of D

$$= \frac{720°}{57} p \quad (p = 3, 4 \ldots . 25)$$

the factors for $n\mu$, $\frac{n}{2}\delta_1$ and $\frac{n}{2}\delta_2$ respectively, viz. :

$$\begin{aligned} &\pm 7{\cdot}285 - 7{\cdot}960 \sin D + 2{\cdot}306 \sin 2D \\ &\mp 3{\cdot}980 + 5{\cdot}537 \sin D - 0{\cdot}637 \sin 2D \\ &\pm 1{\cdot}153 - 0{\cdot}637 \sin D + 1{\cdot}756 \sin 2D \end{aligned}$$

p	Factor for $n\mu_*$	Factor for $\frac{n\delta_1}{2}$	Factor for $\frac{n\delta_2}{2}$	p	Factor for $n\mu_*$	Factor for $\frac{n\delta_1}{2}$	Factor for $\frac{n\delta_2}{2}$
3	+4·7	−1·2	+2·5	15	−5·3	+2·9	−0·5
4	+3·4	−0·3	+2·4	16	−2·6	+1·4	+0·3
5	+2·1	+0·4	+2·0	17	−0·6	+0·2	+0·9
6	+0·8	+1·1	+1·4	18	+0·9	−0·8	+1·1
7	−0·5	+1·5	+0·6	19	+1·7	−1·4	+0·9
8	−1·4	+1·7	−0·1	20	+1·6	−1·6	+0·5
9	−1·8	+1·6	−0·7	21	+1·0	−1·7	−0·2
10	−1·4	+1·1	−1·0	22	0·0	−1·3	−1·0
11	−0·3	+0·3	−1·0	23	−1·3	−0·8	−1·7
12	+1·5	−0·8	−0·6	24	−2·7	−0·1	−2·2
13	+3·9	−2·1	+0·1	25	−4·0	+0·7	−2·5
14	+6·6	−3·6	+0·9				

For full Moon $p = 14\frac{1}{4}$.

Secondly as to Δ_1 Δ_2 ; I put

$$\epsilon = \Delta_1(\cos D - c_1) + \Delta_2(\cos 2D - c_2)$$

when c_1 c_2 are the average values of cos D, cos 2D respectively.

The normal equations are then

$$\Delta_1\Sigma(\cos D - c_1)^2 + \Delta_2\Sigma(\cos D - c_1)(\cos 2D - c_2) = \Sigma\epsilon(\cos D - c_1)$$
$$\Delta_1\Sigma(\cos D - c_1)(\cos 2D - c_2) + \Delta_2\Sigma(\cos 2D - c_2)^2 = \Sigma\epsilon(\cos 2D - c_2)$$

The coefficients on the left-hand side are

$$\Sigma(\cos D - c_1)^2 = \Sigma(\cos^2 D - c_1^2) = \frac{n}{2}\{1 + c_2 - 2c_1^2\}$$

$$\Sigma(\cos D - c_1)(\cos 2D - c_2) = \Sigma(\cos D_1 \cos 2D - c_1c_2) = \frac{n}{2}\{c_1 + c_3 - 2c_1c_2\}$$

$$\Sigma(\cos 2D - c_2)^2 = \Sigma(\cos^2 2D - c_2^2) = \frac{n}{2}\{1 + c_4 - 2c_2^2\}$$

where c_3, c_4 denote the mean values of cos 3D, cos 4D respectively.

My paper in 1903 December enables us to evaluate these quantities as

$$\frac{n}{2}\{1 - 0{\cdot}09 - 0{\cdot}46\} = \frac{n}{2} \cdot 0{\cdot}45 = \frac{n}{2}a$$

$$\frac{n}{2}\{-0{\cdot}48 - 0{\cdot}09\} = -\frac{n}{2}\ 0{\cdot}57 = \frac{n}{2}h$$

$$\frac{n}{2}\{1 + 0{\cdot}06 - 0{\cdot}01\} = \frac{n}{2}\ 1{\cdot}05 = \frac{n}{2}b$$

We shall subsequently require

$$ab = 0{\cdot}47,\ ah = -0{\cdot}26,\ bh = -0{\cdot}60,\ h^2 = 0{\cdot}32,\ ab - h^2 = 0{\cdot}148$$

The normal equations are therefore :

$$\left(\frac{n}{2}\Delta_1\right) + \frac{h}{a}\left(\frac{n}{2}\Delta_2\right) = \Sigma\epsilon\left(\frac{\cos D - c_1}{a}\right) = \frac{n}{2}\Delta'_1$$

$$\frac{h}{b}\left(\frac{n}{2}\Delta_2\right) + \left(\frac{n}{2}\Delta_2\right) = \Sigma\epsilon\left(\frac{\cos 2D - c_2}{b}\right) = \frac{n}{2}\Delta'_2$$

The apparent values Δ'_1, Δ'_2, which may be viewed as arbitrary auxiliary quantities, are defined as above ; and their values have been actually calculated from the formula

$$\Delta'_1 = \frac{2}{n}\Sigma\epsilon\,\frac{\cos D - c_1}{a} = \frac{2}{n}\Sigma\epsilon(\cos D + 0{\cdot}48) \times 2{\cdot}27$$

$$\Delta'_2 = \frac{2}{n}\Sigma\epsilon\,\frac{\cos 2D - c_2}{b} = \frac{2}{n}\Sigma\epsilon\,(\cos 2D + 0{\cdot}09) \times 0{\cdot}94$$

which correspond with sufficient accuracy to the algebraical formulæ.

Then solving for Δ_1, Δ_2 we have

$$\Delta_1 = \frac{ab}{ab-h^2}\Delta'_1 - \frac{bh}{ab-h^2}\Delta'_2 = +3{\cdot}2\Delta'_1 + 4{\cdot}1\Delta'_2$$

$$\Delta_2 = \frac{-ah}{ab-h^2}\Delta'_1 + \frac{ab}{ab-h^2}\Delta'_2 = +1{\cdot}7\Delta'_1 + 3{\cdot}2\Delta'_2$$

The following table is similar to a preceding one :

p.	Factor for $\frac{n}{2}\Delta'_1$.	Factor for $\frac{n}{2}\Delta'_2$.	Factor for $\frac{n}{2}\Delta_1$.	Factor for $\frac{n}{2}\Delta_2$.
3	+2·88	+0·32	+10·5	+5·9
4	+2·54	−0·09	+ 7·8	+4·0
5	+2·11	−0·47	+ 4·8	+2·1
6	+1·66	−0·74	+ 2·3	+0·4
7	+1·16	−0·86	+ 0·2	−0·8
8	+0·66	−0·79	− 1·1	−1·4
9	+0·18	−0·55	− 1·7	−1·4
10	−0·25	−0·20	− 1·6	−1·1
11	−0·61	+0·22	− 1·0	−0·3
12	−0·91	+0·60	− 0·4	+0·4
13	−1·09	+0·88	+ 0·1	+1·0
14	−1·18	+1·02	+ 0·4	+1·2
15	−1·16	+0·98	+ 0·3	+1·2
16	−1·02	+0·76	− 0·1	+0·7

p.	Factor for $\frac{n}{2}\Delta'_1$.	Factor for $\frac{n}{2}\Delta'_2$.	Factor for $\frac{n}{2}\Delta_1$.	Factor for $\frac{n}{2}\Delta_2$.
17	−0·77	+0·41	− 0·8	0·0
18	−0·45	0·00	− 1·4	−0·8
19	−0·05	−0·39	− 1·8	−1·3
20	+0·40	−0·69	− 1·6	−1·5
21	+0·91	−0·84	− 0·5	−1·1
22	+1·41	−0·82	+ 1·2	−0·2
23	+1·88	−0·62	+ 3·5	+1·2
24	+2·34	−0·29	+ 6·3	+3·0
25	+2·72	+0·11	+ 9·2	+5·0

The following table gives the weights of the various quantities when determined from n observations each of weight unity, and also the sum of their values for the 48 periods without regard to sign :

Quantity.	Sum of Numerical Values.	Weight.
μ'	146	n
δ'_1	208	$\frac{1}{2}n$ approximately
δ'_2	131	$\frac{1}{2}n$ approximately
Δ'_1	192	$\frac{1}{2}n \,.\, a = \frac{1}{2}n \div (1{\cdot}5)^2$
Δ'_2	113	$\frac{1}{2}n \,.\, b = \frac{1}{2}n$ approximately
μ	208	$n \times \frac{7\,8\,1}{5\,0\,4\,7} = n \div (2{\cdot}7)^2$
δ_1	235	$\frac{1}{2}n \times \frac{1\,0\,3\,0}{5\,0\,4\,7} = \frac{1}{2}n \div (2{\cdot}3)^2$
δ_2	117	$\frac{1}{2}n \times \frac{3\,2\,2\,8}{5\,0\,4\,7} = \frac{1}{2}n \div (1{\cdot}3)^2$
Δ_1	284	$\frac{1}{2}n \dfrac{ab-h^2}{b} = \frac{1}{2}n \div (3{\cdot}7)^2$
Δ_2	147	$\frac{1}{2}n \dfrac{ab-h^2}{a} = \frac{1}{2}n \div (1{\cdot}7)^2$

Judging from the last eight quantities alone, the probable error of a single observation is $\pm 1''{\cdot}4$. The first two quantities, however, give about double this result, a value which is clearly too large. The weighted mean is $\pm 1''{\cdot}7$ and agrees well with the estimates based upon other analyses.

Taking means for the 48 periods, I obtain

$$\mu = -0''{\cdot}07$$
$$\delta_1 = 0{\cdot}00$$
$$\delta_2 = -0{\cdot}07$$
$$\Delta_1 = -0{\cdot}28$$
$$\Delta_2 = +0{\cdot}10$$

Of these quantities Δ_1 alone exceeds its probable error, and there is no gravitational explanation of a cos D term.

The three quantities μ, δ_1 and δ_2 are liable to probable errors of nearly $0''\cdot10$. Taking, however, the values as given above, the results of my Paper in December require slight corrections, and I now obtain

Correction to tabular semi-diameter	$-0''\cdot47$
Observed Parallactic Inequality	$-126''\cdot46+1''\cdot56=-124''\cdot90$
Observed Variation	$2370''\cdot14+0''\cdot07=2370''\cdot21$

The corrections to the December paper are,

$$\text{for } \mu - 0''\cdot12, \text{ for } \delta_1 + 0''\cdot12, \text{ for } \delta_2 - 0''\cdot04$$

and it will be seen that these numbers are approximately proportional to the factors for day 14. The values in the December Paper are probably slightly erroneous for the reason noted in *Monthly Notices*, December, p. 96, paragraph (i), but the cause there noted will only account for a small half of the discordances. The remaining part is, as far as I know, to be attributed to purely accidental differences. It must be remembered that several corrections have been applied to each observation in the interval between the two investigations. I incline, for the reasons stated, to the belief that the present values are more accurate.

The solar parallax, deduced on the assumption that there is no monthly term in the semi-diameter, has now been raised from $8''\cdot76$ by $+0''\cdot018$ by Professor Brown's calculations (see *Monthly Notices*, 1904 April) and by $+0''\cdot009$ on account of the revised result of the present paper. The value thus becomes

$$8''\cdot787 \pm 0''\cdot007.$$

Solving for $\omega - \omega'$ and $\mathfrak{D}$, I find no considerable terms except

$$+0''\cdot33 \sin \mathfrak{D} \sin D.$$

This is nearly the same term, as I found a month ago, before taking account of the variation. There is no term in cos $\mathfrak{D}$ cos D, and as the coefficient $0''\cdot33$ is only about three times the probable error, I have ceased to think that the term is certainly real.

There is reason to think that the coefficients of sin $(g-g')$ and sin $(g-g'+2\omega-2\omega')$ in Hansen's Tables $(+148''\cdot03$ and $-28''\cdot60)$ require algebraic diminution by about $0''\cdot10$ each, subject to a probable error as large as the quantity itself. Hansen's coefficients are clearly more accurate than Delaunay's.

Newcomb's corrections are now applied in the *Nautical Almanac* in a manner equivalent to

$$N\{1+\tfrac{1}{9}\cos g+\tfrac{2}{100}\cos 2D+\tfrac{2}{100}\cos(2D-g)\}$$

nearly. I have only applied the first two terms $N\{1+\frac{1}{9}\cos g\}$

to the tabular places before 1883. Immediately preceding 1883, $N = -11''$, and the two omitted terms are, for a short time,

$$-0''\cdot2 \cos 2D - 0''\cdot2 \cos (2D - g).$$

There seems to be a very faint trace of this in the values of Δ_2, and a more pronounced trace in the values of Δ'_2 (see periods 110–117), but I find no trace whatever in the coefficients of cos $(2D-g)$ published in a previous paper. This is not unnatural, for quantities of $0''\cdot2$ cannot be obtained from the analysis of a very short period.

On the Comparison between the purely Theoretical and Observed Places of the Moon. By E. Nevill.

The appearance in the last number of the *Monthly Notices* (1903 November) of Mr. Cowell's Note on the Errors of the Moon's Tabular Longitude recalls the result of a more complete comparison between observation and the theory of the Moon as developed by Delaunay's method which I had intended to send to the Society six or seven years ago, but was prevented by reasons no longer existing.

In his interesting note Mr. Cowell remarks that Hansen's *Tables de la Lune* may be brought into much better accord with observation by applying corrections to bring its data into accord with the theoretical results obtained by Professor Hill and M. Radau, and especially by making use of the correction to one of the *Venus* terms of long period suggested by Professor Newcomb. But Mr. Cowell does not give the requisite figures to show clearly to what extent the discord between observation and theory is reduced by these corrections, nor does he give the data for showing distinctly what is still more important, the systematic periodical discordance remaining outstanding.

This omission suggested the abstracting and forwarding to the Society the results of the comparison between pure theory and observation yielded by my earlier investigation, which does clearly show these most important points and afford material for testing some of the conclusions stated by Mr. Cowell in his published paper.

It has been already shown that as far as the perturbations are due to the direct disturbing action of the Sun, there is no systematic difference of any importance between the theory embodied in Hansen's Tables and that which would be obtained from Delaunay's theory.

But when the supplementary terms are considered which arise from the disturbing action of the planets, the figure of the Earth, &c., then there exists grave divergency between the

results obtained by Hansen's method and those obtained by Professor Hill, M. Radau, and others, by an extension of Delaunay's method to the calculation of these terms.

It is to the values determined by this extension of Delaunay's method that the term "pure theory" is here applied. So far as they go, they are accurate; whether they are complete remains to be shown.

Considering my own investigation, the place of the Moon according to pure theory which was to be compared with observation, was obtained from Hansen's Tables by removing the empirical terms, correcting certain of his coefficients which later research had indicated as incorrect, and supplementing these results by certain terms due to the disturbing action of the planets which had been omitted from Hansen's Tables.

The corrections applied by myself differed in some respect from those employed by Mr. Cowell, as will be seen by the following comparison, in which I have followed the notation employed in the first part of this paper.

I. Correction to the secular and semisecular terms:

$$\{-1''\cdot14-29''\cdot17[\mathrm{T}-1800]-3''\cdot86[\mathrm{T}-1800]^2 \\ -21''\cdot47\sin(8\mathrm{V}-13\mathrm{E}+274^\circ\,14')-0''\cdot92\sin(18\mathrm{V}-16\mathrm{E} \\ -g+30^\circ\,12')\}\times\{1+0\cdot1098\cos g\}$$

Presumably, Mr. Cowell, like myself, derived this correction from Professor Newcomb's *Researches on the Motion of the Moon*, page 268 (Washington, 1878), and in the correction to the secular acceleration the figure $-3''\cdot76$ given in his paper is a misprint for $-3''\cdot86$, as given by Newcomb.

II. Corrections to the tabular values of certain terms.

Mr. Cowell employs the corrections *

$$+0''\cdot63\ \cos\Omega$$
$$+0\cdot445\sin\Omega\,.\cos g$$
$$+0\cdot391\sin(2g+2\varpi-\Omega)^*$$
$$-0\cdot62\ \sin(2g-4g'+2\omega-4\omega')^*$$

My own corrections were

$$+0''\cdot400\cos\Omega$$
$$+0\cdot445\sin\Omega\,.\cos g$$
$$+0\cdot391\sin(2g+2\varpi-\Omega)^*$$
$$-0\cdot620\sin(2g-4g'+2\omega-4\omega')^*$$

* In the case of the term depending on the argument cos Ω Mr. Cowell's adopted correction does not seem accurate.

Apparently he derives the correction

$$-0''\cdot63\cos\Omega$$

and

$$+0''.240 \sin (V-E)$$
$$+0''.100 \sin (E-J)$$
$$-0''.100 \sin (2M-E+49°)$$
$$-0''.200 \sin (\quad -g'+2\omega-2\omega')$$
$$+0''.100 \sin (\quad +2\omega-2\omega')$$
$$-0''.185 \sin (\ g \quad +2\omega \quad)$$

III. Additional terms omitted from Hansen's Tables.

Mr. Cowell uses the terms

$$-0''.348 \sin (2V-3E+85°)$$
$$+0''.316 \sin (g+2\varpi-3J+7°)$$
$$-0''.881 \sin (g+2\varpi-2J)$$
$$-0''.681 \sin (g+2\varpi+3V-5E)$$

from the term in the expression of *nz*, on page 8 of the *Tables de la Lune*

$$+7''.76 \sin (\Theta+184° 12')$$

but that is not the complete value in the expression for the longitude, as a portion is derived from the auxiliary function of *s*.

The value really employed by Hansen in his tables is

$$-0''.706 \cos \Omega$$

a value which was reduced on revision in the *Darlegung* to

$$-0''.563 \cos \Omega$$

But, as M. Radau points out, Hansen's tabular value is too large, from his having omitted certain portions of the complete value, and the tabular value should be reduced to

$$-0''.311 \cos \Omega$$

Professor Hill finds the value

$$-0''.293$$

the greater part coming from the effects of the motion of the plane of the variable ecliptic.

Hence the correction necessary to bring Hansen's Tables into accord with theory is very approximately

$$+0''.40 \cos \Omega$$

and not

$$+0''.63 \cos \Omega$$

as adopted by Mr. Cowell.

An examination of the *Darlegung* will show Mr. Cowell that Hansen calculated his adopted value from theory, and it does not in any way depend on the coefficient of the term

$$-0''.97 \cos \Omega$$

deduced by Airy from the observations of the years 1750–1830. (Mr. Cowell attributing it to the value $-1''.06$ deduced from the observation of the years 1750–1851 is an accidental oversight, as that value did not appear till some years after the tables were finished.)

The additional terms employed by myself were

$$-0''.348 \sin (2V-3E+85°)$$
$$+0''.316 \sin (g+2\varpi-3J+7°)$$
$$-0''.881 \sin (g+2\varpi-2J)$$
$$-0''.681 \sin (g+2\varpi+3V-5E)$$
$$-0''.181 \sin (3V-4E+88°)$$
$$+0''.143 \sin (g+2\varpi-20V+19E+272°)$$
$$+0''.112 \sin (2E-4M)$$
$$+0''.206 \sin (\quad 2\varpi-2J)$$

The terms marked with an asterisk are of little importance for the present purpose, where only mean errors for the entire year's observations will be discussed, as their effects will, to all intents, vanish from such means.

The differences between the system of corrections employed by myself and those employed by Mr. Cowell are of little importance, and with the view of obtaining results as comparable as possible with those employed in his investigation I modified my results by striking out the terms not included by Mr. Cowell.

Hansen's Tables, as thus modified, may be regarded as representing the pure theory of the Moon as developed according to Delaunay's method.

And as thus brought into accord with Delaunay's theory they absolutely fail to represent the observations in any degree or for any period.

To bring the modified tables into accord with observation it is absolutely essential to introduce some extra term of very long period and with a very considerable coefficient—a term for which the Lunar Theory, as developed by Delaunay's method, fails to afford the slightest justification.

Mr. Cowell, adopting the suggestion of Professor Newcomb, employs the empirical terms

$$-15''.50 \cos (18V-16E-g) \times (1+2e \cdot \cos g)$$

The adoption of this term as a correction is objectionable, because it is impossible that such a term can exist. There may be one or more terms with periods very similar to that suggested by Professor Newcomb; in fact, there are ; and it is possible that their combined coefficients might be as large as that of Newcomb's empirical term ; but it is impossible that there can exist a term depending on the argument $\cos (18V-16E-g)$ or $\sin (18V-16E-g+90°)$ with a coefficient of fifteen seconds of arc. For even admitting that the neglected higher powers of the disturbing forces might give rise to additional terms sufficient when united to form a combined coefficient of fifteen seconds, the manner in which these extra terms must depend on the terms due to the

lower powers of the disturbing forces prevents any material change in the value of the constant part of the argument. A change of many degrees is impossible.

But as Mr. Cowell has adopted this empirical correction of Newcomb's, and appears to regard it as fairly representing the observations, it has been adopted in the present comparisons, as it was one of the provisional solutions tested in my earlier investigations.

The apparent error of Hansen's unaltered tables was obtained by an extension of the method adopted by Airy in his discussion of the Greenwich observations (*Memoirs R.A.S.*, vol. xxix., 1860), the unknown corrections to the other coefficients being eliminated by their values deduced from the observations of the same period. Airy united his results into nine-year groups in order to eliminate large rapidly fluctuating errors. In my case, as these errors do not exist when the observations are referred to Hansen's Tables, each year was considered independently, in the same manner as developed on pages 392–398 of the memoir on "The Corrections required by Hansen's 'Tables de la Lune'" (*Memoirs R.A.S.*, vol. xlviii.).*

The values as obtained are given as "Errors of Hansen." Then follow the corrections for the terms adopted by Mr. Cowell and specified above. By adding the correction to the preceding error there is obtained the value of the outstanding error of the tables as corrected.

The results are :—

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.		Error of Hansen's Tables.	Cowell's Correction	Outstanding Errors of Tables.
	"	"	"		"	"	"
1750	+ 3·24	+ ·59	+ 3·83	1756	+ 1·93	− 1·31	+ ·62
51	+ 2·47	− ·11	+ 2·36	57	+ 3·10	− 1·49	+ 1·61
52	− ·17	− ·33	− ·50	58	+ 3·29	− 1·03	+ 2·26
53	+ 1·04	− ·80	+ ·24	59	+ 5·04	− ·84	+ 4·20
54	+ ·18	− ·53	− ·35	1760	+ 3·39	− 1·17	+ 2·22
55	− ·28	− ·13	− ·41	61	+ 3·61	− 1·15	+ 2·46

* Owing to the proofs of the Memoir being revised sheet by sheet in Natal from rough notes, whilst the MSS. and remainder of the proofs were in England, it contains a certain number of errors, which in no sense affect the computations or results, but are perplexing as they stand.

The most important is on page 360, where the numerical values of the argument of long period due to Venus have been interchanged.

Minor errors are :

Page 360. Where $a_{1v.}$ is printed for $l_{1v.}$

Pages 374–379. Where a has been printed throughout for $\mathfrak{a}$, where

$$\mathrm{a}' = -\mathrm{a} = \mathrm{A} - \mathrm{A}'.$$

Pages 319, 402, and 404, where the argument of the term depending on

$$f = 2\mathrm{A} - 2\mathrm{A}' + 3l_{,} - 5l_{,,}$$

should be increased by 200°.

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.
	″	″	″
1762	+ 2·14	− ·60	+ 1·54
63	+ 4·52	− ·40	+ 4·12
64	+ ·22	− ·84	− ·62
65	+ 1·30	− ·98	+ ·42
66	+ 2·50	− ·62	+ 1·88
67	+ 3·89	− ·67	+ 3·22
68	+ 1·63	− 1·36	+ ·27
69	+ 2·21	− 1·69	+ ·52
1770	+ ·63	− 1·40	− ·77
71	+ 1·43	− 1·60	− ·17
72	+ 2·84	− 2·25	+ ·59
73	+ 2·40	− 2·47	− ·07
74	+ 2·98	− 2·09	+ ·89
75	+ ·01	− 2·02	− 2·01
76	+ 3·40	− 2·36	+ 1·04
77	+ 3·80	− 2·27	+ 1·53
78	+ 3·21	− 1·61	+ 1·60
79	+ 2·76	− 1·31	+ 1·45
1780	+ 2·01	− 1·54	+ ·47
81	+ 1·39	− 1·41	− ·02
82	+ ·66	− ·77	− ·11
83	+ 1·52	− ·60	+ ·92
84	+ 2·37	− ·99	+ 1·38
85	+ 1·81	− 1·01	+ ·80
86	+ 1·88	− ·60	+ 1·28
87	+ 2·27	− ·62	+ 1·65
88	+ 1·14	− 1·21	− ·07
89	− ·76	− 1·35	− 2·11
1790	− 1·59	− ·99	− 2·58
91	+ ·02	− 1·00	− ·98
92	+ ·99	− 1·46	− ·47
93	+ 2·40	− 1·42	+ ·98
94	+ 1·05	− ·80	+ ·25
95	+ 1·79	− ·56	+ 1·23
96	+ 5·84	− ·74	+ 4·10
97	+ 4·48	− ·43	+ 4·05
98	+ 2·31	+ ·36	+ 2·67
99	+ 3·43	+ ·70	+ 4·03

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.
	″	″	″
1800	+ 2·47	+ ·52	+ 2·99
01	+ 1·87	+ ·70	+ 2·57
02	+ 2·01	+ 1·33	+ 3·34
03	+ 2·09	+ 1·39	+ 3·48
04	+ 4·67	+ ·95	+ 5·62
05	+ 4·26	+ ·90	+ 5·16
06	+ 1·82	+ 1·26	+ 3·08
07	+ 2·65	+ 1·14	+ 3·79
08	+ 2·25	+ ·55	+ 2·70
09	− ·07	+ ·49	+ ·42
1810	+ 1·43	+ ·83	+ 2·26
11	+ 4·48	+ ·90	+ 5·38
12	+ 2·87	+ ·61	+ 3·48
13	+ 2·54	+ ·84	+ 3·38
14	+ 4·35	+ 1·60	+ 5·95
15	+ 5·39	+ 1·90	+ 7·29
16	+ 4·74	+ 1·77	+ 6·51
17	+ 4·99	+ 2·12	+ 7·11
18	+ 3·25	+ 2·83	+ 6·38
19	+ 3·99	+ 2·96	+ 6·95
1820	+ 4·99	+ 2·61	+ 7·01
21	+ 2·24	+ 2·65	+ 4·89
22	+ 2·71	+ 3·07	+ 5·78
23	+ 1·23	+ 2·92	+ 4·15
24	+ ·73	+ 2·31	+ 3·04
25	+ 1·13	+ 2·19	+ 3·32
26	− 1·78	+ 2·47	+ ·69
27	+ ·82	+ 2·28	+ 3·10
28	+ ·91	+ 1·70	+ 2·61
29	+ 2·59	+ 1·77	+ 4·36
1830	+ 2·88	+ 2·27	+ 5·11
31	+ 4·44	+ 2·25	+ 6·65
32	+ 5·01	+ 1·92	+ 6·93
33	+ 1·39	+ 2·19	+ 3·58
34	− ·14	+ 2·80	+ 2·46
35	+ 3·13	+ 2·88	+ 6·01
36	+ 1·76	+ 2·54	+ 4·30
37	+ 1·09	+ 2·69	+ 3·78

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.
	″	″	″
1838	+ 2·93	+ 3·12	+ 6·05
39	+ 2·94	+ 2·93	+ 5·87
1840	− ·23	+ 2·36	+ 2·13
41	− ·87	+ 2·24	+ 1·37
42	− 2·35	+ 2·44	+ ·09
43	− 1·23	+ 2·05	+ ·82
44	− ·85	+ 1·32	+ ·47
45	− 2·62	+ 1·15	− 1·47
46	− 1·00	+ 1·38	+ ·38
47	+ 1·75	+ 1·04	+ 2·79
48	+ ·99	+ ·43	+ 1·42
49	− ·95	+ ·42	− ·53
1850	− 1·51	+ ·75	− ·76
51	− 2·26	+ ·55	− 1·71
52	− 1·55	+ ·05	− 1·50
53	− 1·90	+ ·14	− 1·76
54	− 2·37	+ ·53	− 1·84
55	− 1·32	+ ·34	− ·98
56	− 1·52	− ·27	− 1·79
57	− 2·55	− ·36	− 2·87
58	− 2·42	− ·25	− 2·67
59	− 2·74	− ·76	− 3·50
1860	− 3·42	− 1·70	− 5·12
61	− 3·01	− 2·03	− 5·04
62	− 2·51	− 2·17	− 4·68
63	− 1·30	− 2·88	− 4·18
64	+ ·29	− 3·85	− 3·56
65	+ ·96	− 4·22	− 3·26
66	+ 2·55	− 4·26	− 1·81
67	+ 3·45	− 4·81	− 1·36
68	+ 4·05	− 5·56	− 1·51
1869	+ 3·96	− 5·67	− 1·71
1870	+ 4·65	− 5·50	− ·85
71	+ 4·65	− 5·91	+ ·74
72	+ 7·09	− 6·61	+ ·48
73	+ 8·20	− 6·75	+ 1·45
74	+ 8·89	− 6·72	+ 2·17
75	+ 9·42	− 7·39	+ 2·04
76	+ 9·73	− 8·32	+ 1·41
77	+ 8·99	− 8·75	+ ·24
78	+ 8·20	− 9·03	− ·83
79	+ 9·62	− 9·95	− ·33
1880	+ 10·16	− 11·14	− ·98
81	+ 10·27	− 11·74	− 1·47
82	+ 12·07	− 12·09	− ·02
83	+ 13·25	− 13·07	+ ·18
84	+ 13·94	− 14·21	− ·27
85	+ 14·65	− 14·68	− ·03
86	+ 15·51	− 14·87	+ ·64
87	+ 16·14	− 15·65	+ ·49
88	+ 17·09	− 16·55	+ ·54
89	+ 17·14	− 16·80	+ ·34
1890	+ 17·96	− 16·88	+ 1·08
91	+ 19·60	− 17·60	+ 2·00
92	+ 20·65	− 18·54	+ 2·11
93	+ 20 38	− 18·89	+ 1·49
94	+ 20·02	− 19·16	+ ·88
95	+ 20·45	− 20·15	− ·30
96	+ 20·51	− 21·38	− ·87
97	+ 20·80	− 22·02	− 1·22
98	+ 21·65	− 22·54	− ·89
99	+ 23·80	− 23·75	+ ·05

My original comparison ended in 1894, the few additional years are added now. Summing into the means of groups of nine years :

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.
	″	″	″
1754	+ 1·65	− ·56	+ 1·09
63	+ 2·96	− ·91	+ 2·05
72	+ 1·95	− 1·92	+ ·03
1781	+ 2·18	− 1·28	+ ·90
90	+ ·82	− 1·05	− ·23
1799	+ 2·92	+ ·36	+ 3·28

	Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.		Error of Hansen's Tables.	Cowell's Correction.	Outstanding Errors of Tables.
	″	″	″		″	″	″
1808	+2·71	+ ·85	+3·56	1853	− 1·76	+ ·24	−1·52
17	+4·05	+2·16	+6·21	62	− 1·29	− 2·46	−3·75
26	+1·25	+2·33	+3·58	71	+ 6·26	− 6·10	+ ·16
35	+2·50	+2·59	+5·09	80	+10·69	−10·92	− ·23
44	− ·71	+1·60	+ ·89	89	+17·68	−16·72	+ ·96

It is obvious that the system of correction adopted by Mr. Cowell entirely fails to represent the observed errors of Hansen's Tables; and their application leaves the outstanding errors far larger than before, except for the period 1865–1898. Nor can it be improved by any further permissible alteration in the epoch, motion in mean longitude, or secular acceleration.

In fact, the empirical alteration suggested by Professor Newcomb, though it serves well to represent the observations since 1870, and fairly well agrees with the uncertain observations prior to 1720, fails to properly represent those for the century 1760–1860. When it was suggested, their failure was not so apparent, as it was supposed that Hansen's Tables closely represented the observations between 1750–1850, instead of there being discordances between the tables and observations of several seconds.

Comparing the actual mean errors of Hansen's Tables with the value of Newcomb's empirical correction as given on p. 268 of his *Researches on the Motion of the Moon*, the discordances between them are:

	Error of Hansen's Tables.	Newcomb's Correction.	Outstanding Error.		Error of Hansen's Tables.	Newcomb's Correction.	Outstanding Error.
	″	″	″		″	″	″
1650	−39·3	+40·4	+1·4	1780	+ 1·8	− 1·7	+ ·1
60	−36·5	+37·0	+ ·8	90	+ ·9	− 1·2	− ·3
70	−35·0	+33·6	+1·4	1800	+ 3·0	− ·3	+2·7
80	−31·7	+29·2	+2·5	10	+ 1·9	+ ·5	+2·4
90	−26·9	+24·5	+2·4	20	+ 2·6	+ 1·4	+4·0
1700	−22·3	+19·8	+2·5	30	+ 1·3	+ 1·7	+3·0
10	−14·8	+15·0	+ ·2	40	+ ·1	+ 1·2	+1·3
20	− 9·0	+10·7	+1·7	50	− ·5	+ ·1	− ·4
30	− 6·0	+ 6·7	+ ·7	60	− 1·9	− 2·4	−4·3
40	− 5·0	+ 3·4	−1·6	70	+ 6·0	− 6·1	− ·1
50	− ·4	+ ·9	+ ·5	80	+11·3	−11·2	+ ·1
60	+ 2·5	− ·6	+1·9	90	+18·1	−17·3	+ ·8
70	+ 2·4	− 1·5	+ ·9	1900	+25·0	−24·6	+ ·4

Any system of corrections yielding residuals of from two to four seconds of arc in the modern observations since 1800 cannot

be regarded as other than a complete failure. Mr. Cowell, by diminishing the tabular coefficient of Hansen's Venus term depending on the argument

$$(18V - 16E - g + 30^\circ)$$

by nearly a second, has augmented the discordance between the corrected tables and observation by nearly the same amount.

This failure of Newcomb's empirical correction to represent the observations between 1760–1860 was pointed out on pages 372–373 of *Memoirs R.A.S.*, vol. xlviii., in the discussion of the "Correction required by Hansen's 'Tables de la Lune,'" and it is this failure that swamps the improvement introduced by some of the other corrections employed by Mr. Cowell, which are real corrections which should be introduced into the tables, and do diminish the existing discordances.*

Considering next the correction to the coefficients of the terms depending on the argument, *sine mean anomaly* and *cosine mean anomaly*, deduced by comparing Hansen's Tables with the observations, and eliminating the errors of the coefficients of the other terms in the tabular expression for the longitude, by their values deduced from the observations of the same year, in the same manner as in the memoir on the corrections required by Hansen's "Tables de la Lune" (*Mem. R.A S.*, vol. xlviii. pp. 392–98). These are given under the heading "A_s" for the coefficient of the *sine anomaly* and "A_c" for the coefficient of the *cosine anomaly*, with the sub-heading of "Error of Hansen's Tables."

This is followed by the correction assumed by Mr. Cowell to be applicable on p. 25 of his paper, including the constant correction of $-0''.50$.

Lastly, the errors of the tables after being thus corrected are given under the heading of "Outstanding Errors of Tables."

	A_s.			A_c.		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
1750	+″·97	−1″·62	−″·65	+″·05	−″·53	−″·48
51	+1·65	−1·78	− 13	+1·12	−1·28	− ·16
52	+ ·19	−1·55	−1·36	− ·84	−1·75	−2·59
53	−1·96	−1·03	−3·03	−2·63	−1·89	−4·52

* It must not be supposed that Newcomb's empirical correction is the only available means of reconciling the tables and observations. This would be serious if it were so, as Newcomb's is theoretically impossible. But there are several terms known to exist in the complete expression for the perturbations of the Moon which will represent the existing discordances between the tables and observation with far greater accuracy than Newcomb's empirical correction, by assigning them smaller coefficients than that of Newcomb's terms; coefficients which, if larger than that which can be deduced from the first power of the disturbing forces, are yet not larger than may be found when the complete values of the terms are calculated. In fact there are several combinations of such terms available for the purpose.

	As.			Ac.		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	"	"	"	"	"	"
1754	+1·05	−·39	+·66	−·33	−1·61	−1·94
55	−·14	+·19	+·05	−·19	−1·08	−1·27
56	−·45	+·50	+·05	−1·80	−·49	−2·29
57	−1·67	+·54	−1·13	−1·64	+·10	−1·54
58	+·82	+·32	+1·14	−·22	+·31	+·09
59	+1·44	−·08	+1·38	−·55	+·37	−·18
1760	−1·41	−·43	−1·84	−·89	+·26	−·63
61	−·91	−·54	−1·45	+1·75	+·14	+1·89
62	−2·22	−·72	−2·94	+·81	+·10	+·91
63	+1·93	−·63	−1·30	−2·18	+·22	−1·96
64	−·75	−·56	−1·31	−6·78	+·46	−6·32
65	+·28	−·61	−·43	−2·16	+·66	−1·50
66	+3·97	−·84	+3·13	−3·35	+·71	−2·64
67	+1·81	−1·26	+·55	−·73	+·51	−·22
68	+3·16	−1·68	+1·48	+1·88	+·05	+1·93
69	+2·72	−2·01	+·75	+·72	−·54	+·18
1770	+·75	−2·04	−1·29	−·60	−1·15	−1·75
71	−·65	−1·75	−2·40	−2·47	−1·56	−4·03
72	+2·77	−1·13	+1·64	+2·62	−1·59	+1·03
73	−·45	−·34	−·79	+1·13	−1·27	−·14
74	−·57	+·37	−·20	−1·06	−·62	−1·68
75	+·25	+·96	+1·21	−·33	+·22	−·11
76	−2·39	+1·02	−1·37	−2·51	+·90	−1·61
77	−1·10	+·81	−·29	−·85	+1·34	+·49
78	−·11	+·28	+·39	+1·23	+1·43	+2·66
79	+1·65	−·18	+1·47	−·06	+1·22	+1·16
1780	−·22	−·69	−·91	−2·16	+·94	−1·22
81	−1·18	−·77	−1·95	−·44	+·43	−·01
82	+·01	−·77	−·76	−1·68	+·14	−1·54
83	+·27	−·61	−·88	−2·11	−·02	−2·13
84	+·03	−·42	−·39	−1·04	−·08	−1·12
85	+1·05	−·39	+·66	+·02	−·14	−·12
86	+1·39	−·52	+·87	−2·52	−·32	−2·84
87	+1·90	−·72	+1·18	−1·73	−·67	−2·40
88	−·47	−·98	−1·45	−·85	−1·14	−1·99
89	+1·10	−1·05	+·05	−·88	−1·69	−2·55

	As.			Ac.		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	″	″	″	″	″	″
1790	+ ·11	− ·90	− ·79	+ 2·06	− 1·99	+ ·07
91	− ·31	− ·49	− ·80	+ ·58	− 1·98	− 1·40
92	+ 1·16	+ ·05	+ 1·21	+ ·62	− 1·61	− 1·19
93	− ·42	+ ·60	+ ·18	+ 1·46	− ·90	+ ·56
94	− ·67	+ ·95	+ ·28	+ 1·56	− ·03	+ 1·53
95	− 1·94	+ ·93	− 1·01	+ ·10	+ ·77	+ ·87
96	+ ·69	+ ·51	+ 1·20	− 4·82	+ 1·35	− 3·47
97	− 2·72	− ·12	− 2·84	− 2·53	+ 1·55	− ·98
98	+ 1·66	− ·90	+ ·76	− 4·21	+ 1·33	− 2·88
99	+ ·64	− 1·53	− ·89	− 3·52	+ ·76	− 2·76
1800	+ 3·51	− 1·88	+ 1·63	− 1·52	+ ·30	− 1·12
01	+ 3·59	− 1·94	+ 1·65	− ·49	− ·21	− ·70
02	+ 4·89	− 1·73	+ 3·16	− ·34	− ·48	− ·82
03	+ ·24	− 1·35	− 1·11	+ 4·46	− ·54	+ 4·12
04	+ ·90	− ·98	− ·08	+ ·32	− ·46	− ·14
05	+ 2·24	− ·99	+ 1·15	− 1·48	− ·36	− 1·84
06	+ ·91	− 1·09	− ·18	− 1·36	− ·38	− 1·74
07	+ 2·26	− 1·16	+ 1·10	− 2·59	− ·48	− 3·07
08	+ 2·52	− 1·08	+ 1·44	+ ·37	− ·63	− ·26
09	+ ·94	− ·77	+ ·17	+ 2·41	− ·76	+ 1·65
1810	+ 1·89	− ·26	+ 1·63	+ 1·90	− ·71	+ 1·19
11	+ 1·51	+ ·31	+ 1·82	+ 1·04	− ·34	+ ·70
12	+ 2·24	+ ·78	+ 3·02	+ 1·24	+ ·28	+ 1·52
13	+ ·70	+ ·95	+ 1·65	+ ·45	+ 1·03	+ 1·48
14	− 3·21	+ ·78	− 2·43	+ 4·36	+ 1·88	+ 6·24
15	− ·91	− ·30	− ·61	− 4·34	+ 2·25	− 2·09
16	+ ·20	− ·38	− ·18	− 2·56	+ 2·31	− ·25
17	+ 2·09	− 1·02	+ 1·05	− ·82	+ 1·93	+ 1·11
18	+ 2·49	− 1·46	+ 1·03	− 4·11	+ 1·24	− 2·87
19	+ 1·71	− 1·59	+ ·12	− 1·01	+ ·49	− ·52
1820	+ 2·00	− 1·38	+ ·62	− 1·93	− ·40	− 2·33
21	+ ·44	− ·92	− ·48	+ 2·34	− ·97	+ 1·35
22	+ 1·43	− ·40	+ 1·03	+ 3·62	− 1·20	+ 2·42
23	− 2·01	− ·01	− 2·02	+ ·50	− 1·15	− 65
24	+ ·33	+ ·16	+ ·49	− ·32	− ·98	− 1·30

	As.			Ac.		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	″	″	″	″	″	″
1825	+2·32	+ ·14	+2·46	− ·04	− ·74	− ·78
26	+ ·17	− ·01	+ ·16	+1·89	− ·59	+1·30
27	− ·04	− ·16	− ·20	+ ·39	− ·58	− ·19
28	−2·50	− ·20	−2·70	+4·84	− ·56	+4·28
29	−1·89	− ·07	−1·96	− ·52	− ·52	−1·04
1830	−1·16	+ ·14	−1·02	+3·39	− ·29	+3·10
31	− ·72	+ ·36	− ·36	+ ·28	+ ·13	+ ·41
32	−2·74	+ ·38	−2·36	+1·62	+ ·71	−2·33
33	−1·51	+ ·14	−1·37	−1·85	+1·29	− ·56
34	+ ·23	− ·38	− ·15	−1·69	+1·68	− ·01
35	+3·84	−1·04	+2·80	+ ·06	+1·73	+1·79
36	+1·10	−1·94	− ·84	−1·33	+1·39	+ ·06
37	+1·99	−2·28	− ·24	+ ·52	+ ·73	+1·25
38	+ ·73	−2·35	−1·62	− ·63	− ·10	− ·73
39	+1·51	−2·09	− ·58	+1·05	− ·86	+ ·19
1840	+1·51	−1·53	− ·02	+3·17	−1·36	+1·81
41	·00	− ·81	− ·78	−1·55	−1·50	−3·05
42	+1·50	− ·23	+1·27	+1·30	−1·27	+ ·03
43	−1·00	+ ·15	− ·85	−1·10	− ·78	−1·88
44	−1·89	+ ·14	−1·75	− ·68	− ·20	− ·86
45	+ ·95	+ ·10	+1·05	+1·82	+ ·28	+2·10
46	−1·60	− ·15	−1·75	+1·21	+ ·58	+1·79
47	− ·48	− ·38	− ·86	−1·26	+ ·71	− ·55
48	− ·56	− ·46	−1·02	+1·39	+ ·69	+2·08
49	−1·28	− ·38	+ ·90	+1·14	+ ·70	+1·84
1850	+ ·45	− ·13	+ ·27	−2·47	+ ·51	−1·96
51	+ ·04	− ·05	− ·01	−1·87	+ ·66	−1·21
52	− ·31	− ·07	− ·38	−2·25	+ ·90	−1·32
53	+1·05	− ·30	+ ·75	−1·88	+1·04	− ·84
54	+1·79	− ·71	+1·08	−1·32	+1·00	− ·32
55	+1·23	−1·16	+ ·07	− ·72	+ ·71	− ·01
56	+1·83	−1·49	+ ·34	− ·55	+ ·14	− ·41
57	+2·32	−1·54	+ ·78	+1·46	− ·58	+ ·88
58	+ ·77	−1·21	− ·44	+1·21	−1·29	− ·08
59	+ ·59	− ·67	− ·08	+1·82	−1·76	+ ·06

	As.			Ac.		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	"	"	"	"	"	"
1860	+·08	+·08	+·16	+1·73	−1·88	−·16
61	−·42	+·73	+·31	+1·28	−1·62	−·34
62	−·56	+1·10	+·54	+1·01	−1·09	−·08
63	−·86	+1·17	+·31	+1·88	−·50	+1·38
64	−·66	+·83	+·17	+·67	+·02	+·69
65	−·76	+·30	−·46	+·05	+·26	+·31
66	−·41	−·29	−·70	+·21	+·21	+·42
67	−·20	−·99	−·92	−·04	−·06	−·10
68	+1·22	−1·06	+·23	−·29	−·45	−·74
69	+1·22	−·92	+·16	+·98	−·77	+·21
1870	+·94	−·92	+·02	+·37	−·96	−·59
71	+1·67	−·87	+·90	+·28	−·94	−·66
72	+1·32	−·93	+·49	+·69	−·83	−·14
73	+1·14	−1·12	+·02	+·63	−·77	−·14
74	+2·12	−1·39	+·73	+·85	−·86	−·01
75	+2·26	−1·63	+·63	+2·32	−1·18	+1·14
76	+2·22	−1·69	+·53	+2·43	−1·64	+·79
77	+2·64	−1·45	+1·19	+3·04	−2·09	+·95
78	+·26	−·98	−·72	+2·93	−2·37	+·56
79	+·56	−·34	+·22	+3·26	−2·37	+·89
1880	+·03	+·33	+·36	+3·46	−2·04	+1·42
81	−·62	+·72	+·10	+3·82	−1·44	+2·38
82	−·17	+·79	+·62	+1·54	−·73	+·81
83	+·05	+·48	+·53	+1·07	−·12	+·95
84	+·02	−·09	−·07	+1·19	+·17	+1·02
85	−·45	−·72	−1·17	−·28	+·12	−·16
86	+1·65	−1·25	+·40	−·43	−·29	−·72
87	+1·78	−1·52	+·26	+·20	−·91	−·71
88	+1·97	−1·49	+·44	+1·39	−1·54	−·15
89	+1·88	−1·19	+·69	+1·88	−2·00	−·12
1890	+1·85	−·79	+1·06	+2·29	−2·26	+·03
91	+·34	−·46	−·12	+1·88	−2·31	−·43
92	+·45	−·26	+·19	+4·11	−2·26	+1·85
93	−·91	−·26	−1·17	+3·67	−2·28	+1·39
94	+·43	−·35	+·08	+3·63	−2·41	+1·22

	A_2			A_0		
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.
	"	"	"	"	"	"
1895	+ ·34	− ·41	− ·07	+2·26	−2·73	− ·47
96	+ ·63	− ·34	+ ·29	+3·63	−3·12	+ ·51
97	+ ·98	− ·06	+ ·92	+3·34	−3·85	− ·51
98	+ ·70	+ ·35	+1·05	+4·17	−4·06	+ ·11
99	+ ·66	+ ·80	+1·46	+5·12	−3·96	+1·16

These values exhibit large and rapidly fluctuating variations especially in the earlier period; but it is to only a minor degree that these can be attributed to outstanding errors in the tables. But though by the change of tables there has been eliminated the greater part of the large rapidly fluctuating errors of Airy's values for these coefficients, yet the observed places remain unaltered and retain the full effect of any errors due to faulty reduction.

An examination of Airy's observed places indicates that there is much room for improvement in the reduction of the observations to the adopted observed places, and that they are affected by considerable errors varying with the seasons of the year, which exert exceptional influence on the values deduced for the apparent values of the coefficients of the Moon's *mean anomaly*. Though many of the exceptional values appear due to errors in the adopted elements of reduction of the observations with the transit when its faulty condition was not suspected and its pivots had become much worn, yet the main portion of the irregular fluctuations appear to be due to outstanding errors in the reduction of the quadrant observations. The north polar distances derived from this instrument show many irregular fluctuations, varying with the seasons, which can only be ascribed to uneliminated errors in the adjustment of the quadrant; but there are strong indications of systematic error in the elements used to reduce the observations.

In some instances exceptionally large discordances can be attributed to inexperienced observers having used the instrument for a fraction of the year.

In the absence of all means of referring to the details of the observations—and the printed volumes of reductions afford very little of the requisite data—nothing can be done to eliminate the effects of these irregular or semi-accidental errors, though something could be done to eliminate the effects of the more systematic errors.

An examination of the outstanding errors for the years 1851–1898, when the observations made with the Greenwich Transit Circle are all that can be desired, shows clearly that there remain outstanding considerable periodic errors beyond those eliminated by Mr. Cowell's system of corrections. The fact that

the adopted correction, though in the right direction, is insufficient in amount to eliminate the actual observed error, indicates very distinctly that Mr. Cowell has adopted the right arguments for the terms constituting his system of corrections, but that his coefficients are too small. By increasing all the coefficients of the terms due to the action of the planets by a quarter the observed values would be much better represented.

To better eliminate the errors in the observations themselves, and yet leave the data in a form which enables the character of the outstanding errors to be readily understood, take the mean of every three years in the preceding table of values. The results are—

	As.				Ac.			
	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Outstanding Errors. Nine-year Mean.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Outstanding Errors. Nine-year Mean.
	"	"	"		"	"	"	
1751	+ ·94	−1·65	− ·71	...	+ ·11	−1·19	−1·08	...
54	− ·35	− ·41	− ·76	...	−1·05	−1·53	−2·58	...
57	− ·43	+ ·45	− ·02	...	−1·22	− ·03	−1·25	...
60	− ·29	− ·35	− ·64	−·55	+ ·10	+ ·26	+ ·36	−1·12
63	− ·35	− ·64	− ·99	...	−2·72	+ ·26	−2·46	...
66	+2·02	− ·90	+1·12	...	−2·08	+ ·63	−1·45	...
69	+2·22	−1·91	+ ·31	+·31	+1·67	− ·55	+1·12	− ·49
72	+ ·56	−1·07	− ·51	...	+ ·33	−1·47	−1·14	...
75	− ·90	+ ·78	− ·12	...	−1·30	+ ·17	−1·13	...
78	+ ·22	+ ·30	+ ·52	−·27	+ ·11	+1·33	+1·44	− ·19
81	− ·46	− ·74	−1·20	...	−1·43	+ ·54	− ·89	...
84	+ ·27	− ·47	+ ·20	...	−1·04	− ·08	−1·12	...
87	+ ·94	− ·74	+ ·20	−·17	−1·70	− ·71	−2·41	−1·61
90	+ ·30	− ·81	− ·51	...	+ ·58	−1·88	−1·30	...
93	+ ·02	+ ·53	+ ·55	...	+1·15	− ·85	+ ·30	...
96	−1·32	+ ·45	− ·87	+·06	−2·42	+1·22	−1·20	−1·06
1799	+1·94	−1·44	+ ·50	...	−3·08	+ ·80	−2·28	...
1802	+2·91	−1·67	+1·24	...	+1·32	− ·41	+ ·91	...
05	+1·35	−1·02	+ ·33	+·82	− ·84	− ·40	−1·24	− ·30
08	+1·91	−1·00	+ ·91	...	+ ·06	− ·62	− ·56	...
11	+1·88	+ ·28	+2·16	...	+1·39	− ·26	+1·13	...
14	− ·53	+ ·68	+ ·13	+·98	+ ·16	+1·72	+1·88	+ ·78
17	+1·59	− ·95	+ ·64	...	−2·50	+1·83	− ·67	...
20	+1·40	−1·30	+ ·10	...	− ·20	− ·29	− ·49	...
23	− ·08	− ·08	− ·16	+·25	+1·27	−1·11	+ ·16	− ·07
26	+ ·82	− ·01	+ ·81	...	+ ·75	− ·64	+ ·11	...

	As.				Ac.			
	Errors of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Outstanding Errors. Nine-year Mean.	Error of Hansen's Tables.	Mr. Cowell's Adopted Correction.	Outstanding Errors of Tables.	Outstanding Errors. Nine-year Mean.
	″	″	″		″	″	″	
1829	−1·85	− ·04	−1·89	...	+2·57	− ·46	+2·11	...
32	−1·66	+ ·29	−1·37	−·89	+ ·03	+ ·71	+ ·74	+1·15
35	+1·72	−1·12	+ ·60	...	− ·99	+1·60	+ ·61	...
38	+1·41	−2·22	− ·81	...	+ ·31	− ·08	+ ·23	...
41	+1·01	− ·86	+ ·15	−·39	+ ·97	−1·38	− ·41	− ·13
44	− ·65	+ ·13	− ·52	...	+ ·01	− ·23	− ·22	...
47	− ·88	− ·33	−1·21	...	+ ·45	+ ·66	+1·11	...
50	+ ·59	− ·20	+ ·39	−·11	−1·07	+ ·62	− ·45	− ·08
53	+ ·84	− ·36	+ ·48	...	−1·88	+ ·99	− ·89	...
56	+1·79	−1·40	+ ·39	...	+ ·06	+ ·09	+ ·15	...
59	+ ·48	− ·60	− ·12	+·20	+1·59	−1·65	− ·06	+ ·25
62	− ·69	+1·03	+ ·34	...	+1·19	− ·52	+ ·67	...
65	− ·61	+ ·28	− ·33	...	+ ·31	+ ·16	+ ·47	...
68	+ ·75	− ·92	− ·17	−·03	+ ·22	− ·43	− ·21	− ·07
71	+1·31	− ·91	+ ·40	...	+ ·45	− ·91	− ·46	...
74	+1·51	−1·38	+ ·13	...	+1·27	− ·94	+ ·35	...
77	+1·71	−1·37	+ ·34	+·23	+2·80	−2·03	+ ·77	+ ·89
80	− ·01	+ ·24	+ ·23	...	+3·51	−1·95	+1·56	...
83	− ·33	+ ·39	+ ·36	...	+1·27	− ·23	+1·04	...
86	+ ·99	−1·16	− ·17	+·31	− ·17	− ·36	− ·53	+ ·16
89	+1·89	−1·16	+ ·73	...	+1·85	−1·87	− ·02	...
92	− ·04	− ·33	− ·37	...	+3·22	−2·28	+ ·94	...
95	+ ·47	− ·37	+ ·10	+·29	+3·19	−2·75	+ ·44	+ ·54
98	+ ·79	+ ·36	+1·15	...	+4·21	−3·96	+ ·25	...

It will be seen that the outstanding errors of the three-year groups are considerable and periodical; so that a closer examination shows that, in addition to a term with a period equal to that of the lunar perigee, there are other considerable terms with periods of about four, six, and eleven years. All signs of these disappear when the observations are divided into nine-year groups and the means taken so as to obtain the residuals headed "Nine-year Means." But these latter show very clearly, in turn, that there exist considerable inequalities of about twenty and forty years' period.

It is not a case of tracing one or two missing terms, as Mr. Cowell appears to suggest, but of disentangling and determining the values of at least seven or eight. Some are associated in pairs which, being of similar period, are difficult to determine,

and when deduced from a short period of observations like those for the years 1883–1898 come out as a single term, but with a compound coefficient very different from the theoretical value of either term. Thus Mr. Cowell assumes Radau's theoretical value for the term of the form

$$-0''\cdot68 \sin (g+2\varpi+3V-5E)$$

The observations for the period 1862–1888 yield the values

$$-0''\cdot23 \sin (g+2\varpi+3V-5E)$$

(corrections to Hansen's "Tables de la Lune," *Memoirs R.A.S.*, vol. xlviii.).

In fact if the value of this coefficient be approximately calculated it will be found that the coefficient yielded by the observations is

1896–1882	Coefficient	$= -0''\cdot32$
1881–1866	„	$= -0\cdot20$
1865–1851	„	$= -0\cdot54$

But if the earlier years are similarly treated the values will be found to rise to over $-1''\cdot20$; the mean of the whole series of years from 1751–1896 being approximately

$$-0''\cdot88 \sin (g+2\varpi+3V-5E)$$

It had been my intention to copy out from my early investigation the values of the coefficients of the mean anomaly derived from the expression

$$\text{Observed Errors of Airy's Corrected Table} = a \cos g + b \sin g$$

where Airy's Tables have been corrected for the errors in the coefficients of the terms which in Mr. Cowell's list bear the numbers

1 7 8 10 14 15 16 17 18 21 22 23 24 29
30 39 50 51 53 54 55 56 59 65 66 68 70

all of which have some influence, though in cases only indirect, on the values of the coefficients a and b; instead of merely the three terms 15, 39, 51 used by Mr. Cowell; and so have afforded more accurate data for the basis of investigation. But further consideration makes it doubtful whether these comparisons could be put into a form readily available for comparison with the results obtained by Mr. Cowell by means of his fourteen-month groups. For this reason it has been thought sufficient to give the means for each three-year group, so that the nature of the outstanding errors of less than nine years' period can be better seen before taking the mean of the nine-year groups, from which these irregularities disappear, for comparison with the values given by Mr. Cowell.

Apparently the corrections applied by Mr. Cowell for the "five planetary terms (or figure of the Earth)" are the following :

$$+0''.316 \sin (g + 2\varpi - 3J + 7°)$$
$$-0''.881 \sin (g + 2\varpi - 2J)$$
$$-0''.681 \sin (g + 2\varpi + 3V - 5E)$$

and

$$+1'.562 \sin (A + 30°) \cos g$$
$$+1'.039 \sin ☌ . \cos g$$

No correction seems to have been applied by Mr. Cowell for the effects of the errors in tabular mean longitude.

	b=Coeff. of sin Anomaly.			a=Coeff. of cos Anomaly.		
	Airy's Values.	Cowell's Corr.	Corr. Values.	Airy's Values.	Cowell's Corr.	Corr. Values.
	"	"	"	"	"	"
1751	+2·55	−1·28	+1·27	+ ·03	−1·13	−1·10
54	−1·04	+ ·11	− ·93	+ ·54	−1·07	− ·50
57	+2·45	+1·04	+3·49	−1·07	+1·07	− ·00
60	+1·82	+ ·07	+1·89	+ ·28	+1·39	+1·67
63	−1·86	− ·13	−1·99	−1·15	+ ·88	− ·27
66	+3·32	− ·34	+2·98	− ·05	+ ·74	+ ·69
69	+3·61	−1·51	+2·10	− ·84	− ·80	−1·64
72	+ ·75	− ·63	− ·12	+2·38	−1·61	+ ·77
75	− ·16	+1·36	+1·20	−2·62	+ ·68	−1·94
78	+1·61	+ ·78	+2·39	−2·71	+2·04	− ·67
81	− ·19	− ·27	− ·46	+ ·64	+ ·64	+1·28
84	+2·37	+ ·08	+2·45	−1·24	− ·55	−1·59
87	+1·02	− ·22	+ ·80	− ·39	−1·54	−1 93
90	+1·74	− ·40	+1·34	+1·34	−2·76	−1·42
93	+2·17	+1·10	+3·27	− ·30	−1·24	−1·54
96	−2·07	+1·01	−1·06	−1·52	+1·22	− ·30
1799	+2·95	−1·03	+1·92	−1·18	+ ·41	− ·76
1802	+4·70	−1·23	+3·47	+2·22	−1·61	− ·61
05	+ ·77	− ·49	+ ·28	+1·14	−2·01	− ·87
08	+4·12	− ·58	+3·54	+1·83	−2·27	− ·44
11	+2·88	+ ·81	+3·69	+2·14	−1·53	+ ·61
14	+ ·18	+1·28	+1·46	+1·23	+1·04	+2·27
17	+3·42	− ·52	+2·90	− ·25	+ ·94	+ ·69
20	+2·17	− ·88	+1·29	+ ·45	−2·00	−1·55
23	+ ·12	+ ·49	+ ·60	+3·10	−3·30	− ·20
26	+1·78	+ ·49	+2·29	+2·81	−2·79	− ·02

	b=Coeff. of sin Anomaly. Airy's Values.	Cowell's Corr.	Corr. Values.	a=Coeff. of cos Anomaly. Airy's Values.	Cowell's Corr.	Corr. Values.
	″	″	″	″	″	″
1829	+1·12	+ ·43	+1·56	+3·14	−2·26	+ ·88
32	+ ·42	+ ·88	+1·30	+2·79	− ·50	+2·29
35	+4·79	− ·54	+4·26	−1·12	+ ·52	− ·60
38	+4·29	−1·85	+2·44	+2·14	−1·76	+ ·38
41	+1·11	− ·31	+ ·80	+3·92	−3·70	+ ·22
44	+ ·56	+ ·74	+1·30	+2·62	−2·47	+ ·15
47	+1·68	+ ·12	+1·80	+ ·79	−1·06	− ·37
1850	+1·00	+ ·32	+1·32	+ ·27	− ·34	− ·07

These values show clearly that there are outstanding considerable corrections having arguments of less than twelve years period, besides those considered by Mr. Cowell.

Summing into the means of nine-year groups, so as to eliminate the greater part of the effect of the outstanding terms of less than twelve years' period, there are obtained the results

	b=Coeff. of sin Anomaly corrected.	a=Coeff. of cos Anomaly corrected.		b=Coeff. of sin Anomaly corrected.	a=Coeff. of cos Anomaly corrected.
	″	″		″	″
1757	+1·48	+ ·36	1811	+2·70	+ ·81
66	+1·03	− ·40	20	+1·60	− ·36
75	+1·24	− ·61	29	+1·71	+1·06
84	+ ·93	− ·82	38	+2·50	+ ·00
1793	+1·15	−1·09	47	+1·47	− ·07
1802	+1·56	− ·35			

It is not possible to give for comparison the corresponding values obtained by Mr. Cowell in the absence of sufficiently exact information as to the mean date of his groups.

The two mean differences between the results is that the more completely corrected tables show a much smaller correction to the coefficient of the sine anomaly than the imperfectly corrected tables of Mr. Cowell, or −1″·58 instead of −2″·03, and indicate a decided correction to Damoiseau's mean anomaly as far as can be judged by the use of these coefficients.

Of course these coefficients a and b, as determined from year to year, are not the same as the true coefficients A_c and A_s of the terms depending on the cosine and sine of the *mean anomaly* in the expression for the Moon's longitude, though Mr. Cowell appears to use them as if they were, but are really complex quantities made up of the errors of the *equation of the centre*, *annual equation*, *minor parallactic terms*, *evection*, *parallax*, and a good many other factors which do not disappear from even the nine-year groups. The difference between is clearly shown by the following comparison between the values of the coefficients a

and b and the true coefficients of the terms depending on the *mean* anomaly derived from the observations for the years 1892–1896 by the method detailed in the memoir on the "Corrections to Hansen's 'Tables de la Lune'" (*Memoirs R.A.S.*, vol. xlviii. pp. 301–310).

	Coefficient of sine Anomaly.		Coefficient of cosine Anomaly.	
	b	A^s	a	A^c
	"	"	"	"
1892	− ·96	+·45	+3·90	+4·11
1893	−1·58	−·91	+2·88	+3·67
1894	− ·24	+·44	+2·98	+3·63
1895	− ·23	+·34	+2·01	+2·26
1896	− ·18	+·63	+2·60	+3·64

Mr. Cowell employs the value of the coefficient a as if it truly represented the value indicated by the observations of the coefficient of the mean anomaly on the expression for the Moon's longitude, and suggests that the application of Newcomb's empirical term

$$-15''{\cdot}50 \cos (A) (1 + 2e \cos g)$$

to the expression for the Moon's longitude would reconcile the observed and calculated values, or approximately

1750	$a = +0''{\cdot}41$	$(-1''{\cdot}72 \cos A) = +0''{\cdot}86$	corrected value $= +1''{\cdot}27$
1800	$= -0{\cdot}45$	$= +1{\cdot}70$	$= +1{\cdot}25$
1850	$= +0{\cdot}50$	$= +0{\cdot}58$	$= +1{\cdot}08$
1891	$= +2{\cdot}16$	$= -0{\cdot}96$	$= +1{\cdot}20$

This result if sound would indicate that Damoiseau's epoch of mean anomaly was about 12″ too large, but its secular motion nearly correct. Mr. Cowell apparently regards this as some confirmation of the accuracy of Newcomb's empirical correction.

But the result can only be regarded as satisfactory by disregarding the enormous discordances between theory and observation which are introduced by these assumptions employed by Mr. Cowell. This is clearly shown by the following comparison between the observations and Damoiseau's Tables after correcting them so as to bring the coefficient of the perturbations by the Sun into accord with theory. The table shows, first, the results of the comparison with Damoiseau's Tables without any long-period term; secondly, after the addition of the theoretical value of Hansen's direct Venus term with the argument sin (A + 30°); and thirdly, the result after adding also Newcomb's empirical term with the argument cos A.

	Damoiseau. (D−Obs.)	Damoiseau. +14·4 sin (A+30°)	Damoiseau. +14·4 sin (A+30°) −15·5 cos A		Damoiseau. (D−Obs.)	Damoiseau. +14·4 sin (A+30°)	Damoiseau. +14·4 sin (A+30°) −15·5 cos A
	″	″	″		″	″	″
1750	+3·0	+10·4	+17·9	1830	− ·1	−14·0	− 2·9
60	+2·5	+ 6·9	+17·4	40	− ·0	−14·4	− 6·1
70	−1·0	+ ·2	+14·9	50	−1·7	−15·9	−10·8
80	−3·5	− 5·6	+ 8·8	60	−5·7	−18·9	−17·3
1790	−4·4	− 9·7	+ 5·6	70	−2·4	−13·8	−15·8
1800	−1·5	− 9·7	+ 5·7	80	−3·5	−12·7	−18·1
10	− ·1	−10·8	+ 4·0	1890	−5·1	−11·5	−20·1
20	+ ·9	−11·5	+ 1·8				

That Damoiseau's tabular value for the mean anomaly does not represent the observations is shown by the following comparison between the observations and Damoiseau's Tables as corrected.

A_c = Value of Coeff. of cosine Anomaly.

	(D−O)	Cowell's Corr.	Damoiseau corrected.	Airy's Values.
	″	″	″	″
1750	+1·15	−1·21	−0·06	—
60	+1·06	+ ·90	+1·96	+ ·34
70	+1·10	−1·54	− ·44	− ·46
80	+ ·04	+1·60	+1·64	+ ·16
1790	+1·05	−2·22	−1·17	− ·30
1800	− ·92	+ ·57	− ·35	+ ·61
10	+2·91	− ·87	+2·04	+1·30
20	+1·53	− ·10	+1·43	+1·23
30	+3·83	− ·31	+3·52	+2·04
40	+2·98	−1·70	+1·28	+2·58
50	+1·55	+1·21	+2·76	+1·80
60	+3·69	−2·52	+1·17	—
70	+2·03	+ ·74	+2·77	—
80	+4·35	−1·57	+2·78	—
90	+2·91	+ ·74	+3·65	—

These results are in accord with those obtained by Airy, whose results are approximately corrected to reduce them to the same basis, only using the correction for the terms depending on the Moon's node derived from observation. The improved consistency due to using the larger values yielded by observation for the terms with periods close to that of the node is marked.

The improvement in the values deduced by Mr. Cowell by applying Newcomb's empirical term simply means that having

thrown the tabular values into discord with observations by applying to the tables the term

$$+1''\cdot562 \sin (A+30^\circ) \cos g$$

to rectify the evil, and remove the discordances thus introduced, and to neutralise the effects of Damoiseau's erroneous value for the motion in mean anomaly, he has been obliged to introduce the consequent empirical term

$$-1''\cdot722 \cos (A) \cos g$$

for which theory affords no justification.

The only satisfactory value for the coefficient of the cosine mean anomaly is that obtained by eliminating the errors in the tabular value of the mean longitude by means of their observed errors. If that is done in the case of the observed coefficients of Damoiseau's mean anomaly, the apparent agreement at the different epochs at once disappears, and the need for introducing a considerable correction becomes manifest. Nor can it be assumed that the corrections which ought to be introduced for the errors of the tabular longitude are obviated by the existence of corresponding opposite errors in the tabular longitude of *perigee*; because if that were so, corresponding symmetrical errors would be found in the coefficient of the cosine of the evection, and these do not exist.

There is only one further point to which I would draw Mr. Cowell's attention, and that is to the value assigned by him to the correction required by Airy's value of the principal figure of the Earth term. Mr. Cowell combines this term with that due to the *nutation*, and apparently deduces a correction to the combined coefficients by comparing them with the sum of the values assigned respectively by Hill and Hansen to these terms.

Thus

Figure of Earth.		Nutation.		Sum.
Airy	$= +6''\cdot60 \sin \Omega$	Airy	$= -16''\cdot78 \sin \Omega$	$= -10''\cdot18 \sin \Omega$
Hill	$= +7\cdot67 \sin \Omega$	Hansen	$= -17\cdot33 \sin \Omega$	$= -9\cdot66 \sin \Omega$
			Difference ...	$= +0\cdot52$

This difference, which apparently Mr. Cowell takes as being only $+0''\cdot45$, is adopted as the correction required by Airy's value.

But on reconsideration Mr. Cowell will see that this requires revision. For Airy determines the observed R.A. of the Moon by direct comparison with the apparent R.A. of certain standard stars in deducing whose apparent place he uses the value of the nutation employed in the *Tabulæ Regiomontanæ*. As this is $= -16''\cdot78 \sin \Omega$, he is bound to use the same value for the

place of the Moon, and any error in the adopted value of the nutation practically disappears from the result. This point has been already dealt with by Airy in his *Reduction of the Greenwich Lunar Observations* (1831–1851), p. xxiv.

Hence the correction to be applied to Airy's value is the difference between Airy's value and Hill's value for the figure of the Earth Term or

$$+1''\cdot07 \sin \Omega$$

and not

$$+ \cdot45 \sin \Omega$$

the value employed by Mr. Cowell.

By greater skill and ingenuity Mr. Cowell may be more successful than myself in dealing with the investigation of the corrections required by the lunar tables in the rather fragmentary manner he has adopted. I failed, and I fear that further experience will lead him to the same conclusion as myself—that no successful result can be expected without taking the problem up as a whole and simultaneously taking into consideration the inequalities of all kinds and periods.

Natal Observatory:
1904 *January* 21.

Photographic Chart of the Heavens to Argelander's scale $1^\circ = 20$ *mm., with special reference to the Milky Way.* By J. Franklin-Adams; *with description of Lenses and Mounts* by H. Dennis Taylor and Alfred Taylor.

Preliminary Notes by J. Franklin-Adams.

To Sir David Gill am I indebted for the suggestion of photographic research into the structure of the Milky Way. The tentative and experimental method by which the work has been carried out may be interesting in detail.

During the winter of 1897–8 Mr. Dennis Taylor, of Messrs. Cooke & Sons, was asked to design a 6-inch lens of short focus and considerable field-focus, $\frac{f}{4}$. During the autumn of 1898 the lens was temporarily mounted upon an 8-inch German equatorial, also by Cooke & Sons: it gave fair definition over a field, $20^\circ \times 20^\circ$ upon a plate 10 in. × 10 in., working with full aperture. This was in the observatory at Machrihanish House, Argyllshire, with the conveniences of exact time, electric current, developing rooms, and workshops. After working for a

year upon various problems of diffusion of focus, speed, diaphragm, exposure, &c., I had the good fortune to receive a month's visit—in January 1900—from Professor Barnard, who had heard of the lens, and had travelled to Machrihanish to examine it. After scrutinising the results to date he worked for ten nights at experiments of his own, especially comparing the lens with his celebrated Petzval as to rapidity. For this purpose we made a rough camera for his lens and placed the two side by side.

We then journeyed together to York, taking with us a set of our photographs—upon patent plate—mostly of *Orion*, as giving stars of many magnitudes, and two important nebulæ. After discussion with Mr. Dennis Taylor it was decided to recompute curves and alter the ratio to $\frac{f}{4{\cdot}5}$, thus reducing the field, and make a new experimental lens of 4 in. diameter upon these new lines; if successful the 6 in. to be refigured in accordance with experience gained in making the 4 in.

This 4-in. lens was finished in time to take to Santa Pola in Spain with the Scottish Eclipse (1900 May 28) Expedition, to which I was attached; some photographs of the corona were taken with it, and, after the eclipse, also some of stars, *Jupiter*, and the Moon. The results were excellent, the plates were taken to Mr. Dennis Taylor, and the re-figuring of the 6 in. put in hand.

The experiments with the 6-in. lens in its original state were now repeated with the lens in its altered form, and were so successful that a 10-in. lens was ordered, a detailed description of which, by Mr. Dennis Taylor, is appended to these notes. I would wish to say that I alone am responsible for the choice between a field of $12° \times 12°$ with good images everywhere, or a field of $15° \times 15°$ with the images at the corners only "fair." I chose the latter partly because it would reduce the magnitude of the undertaking before me and partly because 120 of the 206 plates would each cover exactly an hour of R.A.

On receiving the 10-in. lens I found that its weight in brass and aluminium mount was about 100lb.: it was mounted temporarily for experimental work on the 8-inch equatorial with 4-inch guider, and prints were sent to Professor Kapteyn, of Gröningen, and, in consideration of his admirable work on the Milky Way, to M. Easton, of Brussels. Professor Kapteyn gave much kindly advice, criticism, and approval of the work. In one square (equatorial) degree in *Cygnus* he counted 670 stars—exposure 2 hours. This gives in a space $2° \times 2°$ more stars than are visible to the naked eye in a hemisphere. The exposure adopted was 2 hours 20 minutes for British stations, 2 hours for southern stations.

Mount.—After visits to every important observatory in Great Britain and to several on the Continent—and I would especially acknowledge great kindness from Dr. Wolf, of Heidelberg, and Professor Becker, of Glasgow, and from all the staff of

Potsdam—I decided upon an English mount with modifications. A description of this, by Mr. Alfred Taylor, of Messrs. Cooke & Sons, follows this paper. I will only mention the points for which I am specially responsible. (*a*) Two 6-in. guiding telescopes instead of one, (*b*) a new eyepiece carrier for rapid finding of guiding stars and fine adjustment when found. As the instrument will in charting always be set to an arbitrary point, *e.g.* 0^h–60°, there may not be, and as a fact there generally is not, a catalogue star in the field. When only small stars are available the colour decides which shall be used. (*c*) Against much good advice I chose a circle instead of a sector, and a circle for slow motion in declination, driving both slow motions by motors. I do not know whether I can lay claim to being the first to use motors for these controls, but they have been successfully used by me for some years. Especially is it useful in alteration of declination necessitated by the varying refraction, the touching of the telescope in second-controlled clocks often causing a dropped second. With the luck of varying atmospheric densities happening at the right time and in the right direction, some plates have been run by the Repsold clock for the two hours without hand interference. (*d*) An arrangement by which either of the guiding telescopes can be thrown 10 degrees or less away from the direction of the camera. By this means, photographs of comets can be guided by the head in the corner of the plate, thus utilising the whole of the plate—15 to 23 degrees—for the tail. I am indebted for this suggestion to my friend Professor Barnard. (*e*) I made a full-sized schematic model of the mount and a small model of the house, which latter Messrs. Cooke developed and copied.

Expedition to the Cape.—In 1903 June my assistant, Mr. G. N. Kennedy, sailed for Cape Town, taking with him the complete instrument and its house. I followed five weeks later, found the house erected and the piers built, and within a fortnight of arrival all was ready for final adjustment of mount and for focussing and centering of the lenses and réseaus. The réseau for the 10 in. was the only exception to all being in order: it had been engraved and packed whilst I was ill in bed, and unfortunately some lines, through not being deeply engraved, are missing upon the early prints 19^h to 23^h, after which a new réseau arrived. It is immaterial, however, as transparent scales are to be used with the proposed atlas, with the degrees of declination marked exactly in hyperbolas or approximately in circles as may be decided; whilst for counting stars on the negatives the measuring machine can be adapted to bridge the missing lines. There were personal reasons why it was convenient to finish the charting of the southern hemisphere first, but the following extract from M. Easton's work, *Distribution de la Lumière Galactique*, is perhaps sufficient without these:

"La distribution des grandeurs stellaires trouvée par Kapteyn n'est pas en contradiction avec la supposition que nous avons faite.

Ce serait un travail intéressant de pousser plus loin cette comparaison, mais je n'ose décider si les données fournies par la présente étude ne sont pas trop incomplètes, presque rien n'étant connu sur la distribution des étoiles dans l'hémisphère austral, ni si elles comportent la précision nécessaire pour des recherches pareilles."

The equatorial plates were compared and identified with Argelander and Schönfeld, but comparison with Gould for the plates south of these was hopeless. For this reason and for the count of the lucid stars I fell in with Sir David Gill's suggestion to take also "triangle plates" with the 10 in. lens. The working scheme is therefore as follows :

91	plates	from	$+7° 30'$ to $+90°$
91	„	„	$-7° 30'$ to $-90°$
24	„	„	$+7° 30'$ to $-7° 30'$
206	„	„	15 in. × 15 in., 2 hours.
206	„	„	15 in. × 15 in., 7 minutes ; 3 times in triangle form.
206	„	„	12 in. × 10 in., 2 hours.
618			
618	transparencies.		
1,236	plates.		

The scheme for the Cape includes :

91 + 24 = 115	plates	15 in. × 15 in., 2 hours'	exposure.
115	„	15 in. × 15 in., 7 mins. 3 times	„
115	„	12 in. × 10 in., 2 hours'	„
345			
345	transparencies.		

Total 690 plates.

Besides these there will be about fifty duplicates, and some picture plates of long exposures—four to five hours.

Each 15 in. × 15 in. is compared with its fellow triangle plate, and with the 12 in. × 10 in. of the same region.

When the 10-inch proved such a success, I at once communicated with Messrs. Cooke as to a duplicate, but, alas ! the melting from which my discs were made had all been used, and another melting could not be depended upon to give the same focus and therefore the same scale. I decided therefore to mount the 6-inch for duplicate plates. If I had been able to arrange for twin lenses of 10 in. I should not have required the transparencies for sending home in a separate steamer, and should

have saved considerable time in being obliged to take several exposures of 15 in. × 15 in. and 12 in. × 10 in. alone for spoiled plates. Duplicate plates are absolutely necessary for identifying nebulæ, perhaps a nova, meteors, faults of réseau, or blemishes in film.

The scale of the 10-in. lens is 20 mm. = 1° on the equator
" " 6-in. " 11 mm. = 1° " "
this latter only approximately.

The plates of the 10 in. are 15 in. × 15 in. (approximately $0^m{\cdot}381$)
" " 6 in. " 15 in. × 10 in., of which only 10 in. × 10 in. will be used, and these will give a good overlap, which the large plates do not.

Both lenses are worked at full aperture.

The good and fair field on each plate is 15° × 15°.

The developer throughout is amidol; at some important observatories the labour of development is much reduced by *time* and not by *visual* development. The developer is applied at a certain strength and temperature for a given number of minutes. The plates under consideration have been developed visually. When safe, ample, and equally distributed light is not available this time system has much to recommend it, but it would probably mean a fresh developer for each plate.

The chart plates are Cadett Lightning; the triangle plates are Imperial flash-light; the transparencies are ordinary plates of both makers.

The driving of the instrument in R.A. by a Repsold clock is a great success. Some plates have been exposed for the two hours without once touching the R.A. motor, but as a rule the varying refraction in so long an exposure and so large a field makes a slight alteration, say three or four times an hour, both in R.A. and decl. advisable. It is quite remarkable that with the varying driving power required in different positions—owing again to the long exposure—this clock drives steadily on with hardly any variation of pace, whereas a Russell Mouse-control clock would stop. If, too, as occasionally happens, a little grit in the driving worm or some other unknown cause should slow down the motion, it is only a slight slow down, and not the miss of a whole second of time as in controlled clocks. The circle instead of sector—a particularly fine one—is a great convenience.

I have been asked why I have deviated from my first intention of photographing the Milky Way only. The reply is (*a*) that the ramifications and extensions of the Milky Way are so undefined that it seemed advisable to chart all regions; (*b*) that as nine months' work, approximately, would be required for the Milky Way alone the whole of the hemisphere could be charted in the same time, although with, of course, more work.

Many points—*e.g.* length of exposures—have been decided because of their bearing on the Milky Way. The intention is to publish the work as an atlas in triplicate *form*: (1) the

15 in. × 15 in.; (2) the 12 in. × 10 in. as a check upon false nebulæ and false stars, both upon paper with gelatine surface; (3) the triangle plates upon writing paper, and for this reason, besides being a second check upon faults in film, they should be a valuable help to workers in special branches of stellar work. A double-star worker, for instance, could number all known double stars according to the several catalogues, S 1004, B 191, I 34, and so on. The same with variables, nebulæ, &c. The expense of such publication would, however, be so serious that I must wait until better times in South Africa for a millionaire to help me with the cash, and in the meantime I propose to print photographically about five copies for public institutions. Any printing, whether photographic or mechanical, will, however, be absolutely *untouched*, and will show all faults, scratches, &c. It seems to me that a single interference of only one spot in a thousand plates would cast a doubt upon the pictorial integrity of any given region.

To record my sense of thanks to Sir David Gill for his kindness in allotting me a splendid site, and for his ever-kindly help and advice, is a great pleasure. Each one of the staff of the Royal Observatory at Cape Town has done his best to render the work as easy and as agreeable as possible.

Professor Barnard has had a 10-inch lens and mount made in America, the lens by Brashear and the mount by Warner & Swasey. It will be interesting to compare the two instruments. I have reason to think that of the two his will work upon a rather smaller field, and that his lens will be more rapid. I do not know the direction in which he proposes to work, nor do I know the scale.

In these days of many observatories in Great Britain, understaffed though they may be; of many more on the continent of Europe, supplied liberally with Government funds; and of still more in America, where wealthy citizens give large sums to build, house, and endow instruments of record size, it may seem difficult to find an unoccupied field of research. That there are such fields, however, is certain, and among them it seems likely that many will be found in that domain which contains the secrets of the structure of the universe.

Description of the Lenses. By H. Dennis Taylor.

Mr. Franklin-Adams has asked me to give a condensed description of his 10-inch aperture Cooke lens, with a short explanation of the theory of its construction.

It is a particular case of the Cooke photographic lens modified for celestial purposes. The essential aim of the Cooke lenses was to obtain a flat image substantially free from astigmatism, besides being rectilinear and achromatic, with the minimum number of lenses possible. This result was obtained with only three simple lenses, two outer positive lenses and an inner

negative lens, which latter had to perform simultaneously the following functions:

1st. To correct the spherical aberration of the two positive lenses, on the axis as well as obliquely.

2nd. To correct the chromatic aberration of the two positive lenses, not only along the axis but obliquely, and also be so placed in position as to cause the final images in various colours to be of the same size and free from curvilinear distortion.

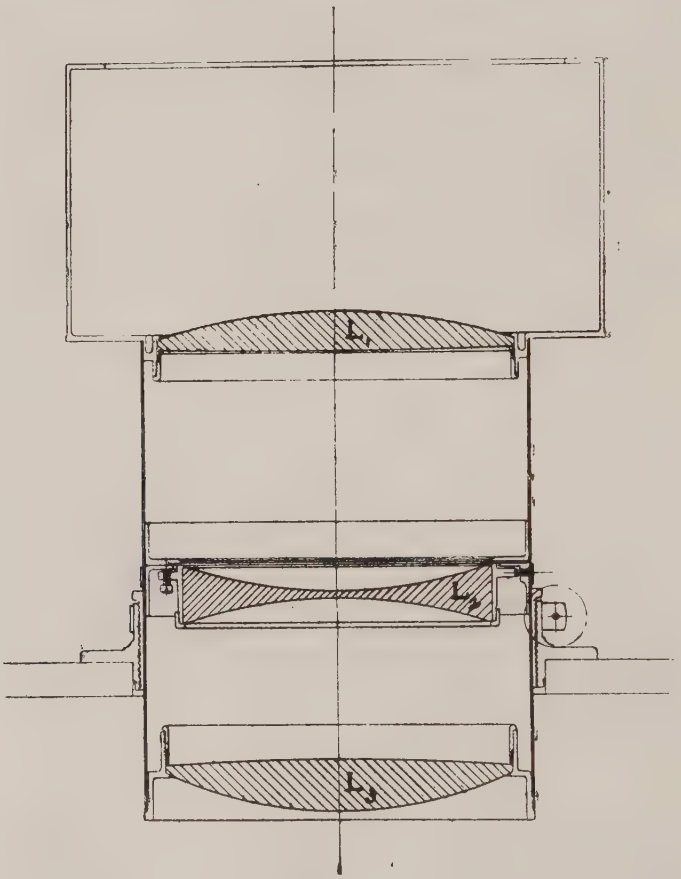


Fig. 1.

3rd. To correct the curvature of image of the two positive lenses, if possible both in primary and secondary sections of the same oblique pencil simultaneously, so as to bring about a final flat image free from astigmatism.

4th. To correct the coma, if any, produced in the oblique pencils by the two positive lenses.

All the above corrections may be carried out simultaneously, although very much interlocked, by a suitable choice of materials, relative powers, and shapes of the lenses, together with suitable separations. A longitudinal section of the complete lens is given in fig. 1.

Spherical Aberration.

It is obvious that we have the power of varying the collective spherical aberration of the two positive lenses between the extreme values $(a+a)$ and 2^3a or $8a$ for any constant aperture, combined power, and separation between the two positive lenses. If they are of equal powers and equal aberrations we have the sum of both aberrations or $(a+a)$, whereas if the power of one lens is about vanished and the power of the other is correspondingly about doubled then the collective aberrations become $(0+8a)$. Thus the collective spherical aberration is capable of being varied to suit any desired negative lens.

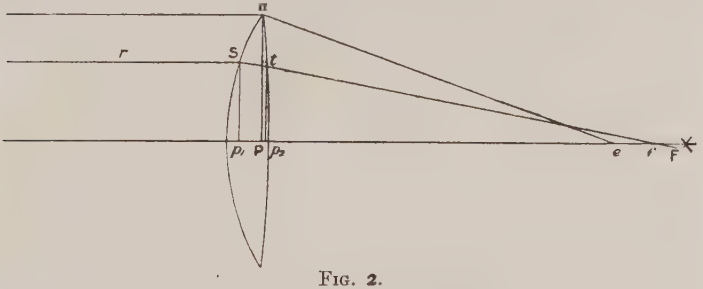


FIG. 2.

There remains for lenses of large relative aperture the question of zonal aberration, a defect of no particular consequence for camera lenses, but of immense importance in the case of lenses for celestial photography which are required to give the sharpest possible images of stars.

Fig. 2 represents a double convex lens on which falls an axial pencil of parallel or divergent rays.

Let the curves be carried out to a sharp edge a, and let the semi-aperature aP (a—P being perpendicular to the axis) be called A. Let a ray be traced through the sharp edge. It meets with a certain amount of spherical aberration, which is a function of A^2, and is refracted to the point e considerably nearer to the lens than F, the point where the ultimate axial rays come to focus. Then e—F is the longitudinal spherical aberration. Let an intermediate ray r—s—t—f be considered. It strikes the first surface at s at a perpendicular distance s—p_1 $(= y)_1$ from the axis and it meets with a certain amount of spherical aberration, which is a function of y_1^2. After refraction the ray converges

towards the axis, and after traversing the thickness of glass $s-t$ it meets the second surface at t nearer the axis, so that $t...p_2$ or y_2 is appreciably less than y_1. It is this disparity between the two y's at the two surfaces at positions in the lens between the axis and the sharp edge which gives rise to zonal aberration.

If the aberration of such a lens is examined by projecting the sections of the cone of rays on to a screen or by an eyepiece then the zone of aberration is so disguised by the much greater general aberration as to be invisible ; but supposing the general aberration is neutralised by the general aberration of a negative lens, then the zonal aberration shows up, and we get an imperfect focus ; for while we may get the extreme edge ray to focus at the same point as the ultimate axial rays, yet there is a zone in the lens from which the rays cut the axis at a point considerably inside of the focus for edge and centre rays. If a section of the cone of rays be taken at a point a little inside of that focus the patch of light takes the form of a disc with a bright zone about half-way between the centre and the edge, while outside of the focus the edge and centre of the disc of light are the brightest. If Y stands for either y_1 or y_2 then the zone of aberration is a function of $Y^2(A^2-Y^2)$ into the spherical aberration of the first surface, and obviously is at a maximum when $A^2 = 2Y^2$, which occurs round a zone of the lens where the thickness is half of the central thickness, the radius of the zone being $\frac{A}{\sqrt{2}}$. It is obvious then, that if we could always use lenses whose thicknesses at the edges were more than half of their central thicknesses, their aperture would always be inside their zones of aberration, and the latter be practically got rid of ; but large thicknesses are so objectionable on account of weight and light absorption, especially in the case of large lenses for stellar photography, that it is scarcely to be thought of, and other methods have to be devised. We have to deal with the case of the rays impinging upon the first lens being parallel, and those impinging upon the third lens being divergent. In the case of the first lens the zone of aberration is a function of the aberration of the first surface, and (supposing shape of the lens is kept constant) that aberration is a function of $\left(\frac{1}{r}\right)^3 \left(\frac{1}{F}\right)^2$, or the cube of the reciprocal of the radius of curvature into the square of the focal power.

Therefore the zone of aberration of the first lens can be enormously reduced by keeping down its power, while, since in the back lens the rays traverse its substance more nearly in a state of parallelism than in the case of the first lens, an increase in the power of the back lens balancing the decrease in power of the front lens does not augment the zone of aberration of the back lens in anything like the same degree. For this reason the power of the back lens was made preponderant. In the first model the two positive lenses were equal in power, and the zone of aberration was so bad that the lens was practically useless for

stellar work in large sizes, and at any aperture over $\frac{F}{5}$, nor could the zone of aberration be got rid of by special figuring, whereas the present arrangement of making the power of the back lens about $1\frac{1}{2}$ times the power of the front lens so reduces the zone of aberration as to render it possible to get rid of it entirely by special retouching ; a somewhat troublesome but quite practicable process. In the case of negative lenses which grow thicker as the centre is left and have no sharp edge it is obvious that zones of aberration do not arise.

Of course the state of the chromatic correction of the final image along the axis is simply a matter of the mutual adjustment of the relative dispersive powers of the positive and negative lenses.

But whether or not the oblique image of stars shall also be achromatic, or whether they shall be drawn out into spectra, pointing to the centre of the field, depends upon the negative lens being placed at a certain position between the two positive lenses. Should this condition be fulfilled then the other highly important condition of freedom from distortion or rectilinearity is simultaneously attained.

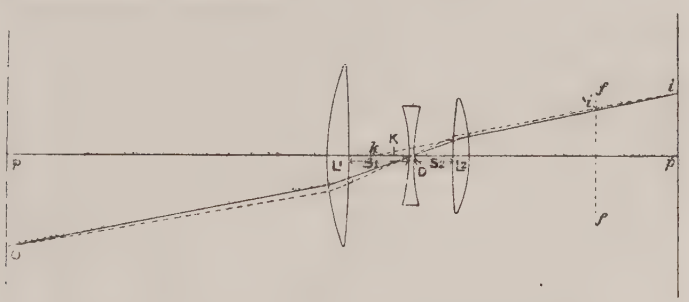


Fig. 3.

The diagram fig. 3 shows one of these lenses considered as reducing or copying in the ratio 3 to 2. OP is the original and *ip* the image, O being a particular point in OP and *i* its image. As the two positive lenses are of the same shape, but turned in opposite ways, and the distance $PL_1 : pL_2 :: F_1 : F_2$, and we place the diaphragm point at D, such that $L_1D : L_2D :: F_1 : F_2$ also ; then it is obvious, without stating the formulæ for distortion (which fully confirm this case), that all is symmetrical on each side of D and $ip : OP :: L_2p : L_1P$ exactly. Hence there is no distortion, and the lens gives a rectilinear image. Moreover, all the variously coloured oblique principal rays from O will pass through D, although traversing each lens L_1 and L_2 at different distances from the axis. For instance, the solid line shows the path of a yellow principal ray from O, while the dotted line shows the path of a blue principal ray from O. If, therefore, the centre of the

negative lens, of properly chosen dispersive power, is also placed at D, the intercrossing or diaphragm point, then we secure that the various colours come to final focus in the same plane, while at the same time the conditions of oblique achromatism also hold good, for the variously coloured principal rays go straight through the centre of the negative lens as if it were not there, and traverse L_1 and L_3 at heights from the axis always proportionate to their focal lengths. Hence perfect symmetry prevails. But supposing the original OP is removed to a much greater distance or infinitely far off, and p moves up towards the principal focal plan ff, then the conditions of oblique achromatism for the image of the point O no longer strictly hold good. It is obvious that the blue image of O at i will fall a little further away from the axis than the yellow image at the same time that positive distortion will arise. For the image of the diaphragm point D as viewed through the back lens L_2 is subject to spherical aberration, the principal ray or line of projection ik radiating from k, whereas the image point of D formed by L_2 would be at K were it not for its spherical aberration. Thus, k—K is the longitudinal spherical aberration of L_2. Therefore the line of projection k—i strikes the principal focal plane f—f too far from the axis, and there is positive or pincushion distortion of the images of straight lines. In order to correct this it is necessary to move the negative lens nearer to L_1 by such an amount as is necessary to again make the various coloured rays from O or a star all strike the principal focal plane f—f at the same distance from the axis, when it will be found that the lens will also become quite rectilinear again. In the case of Mr. Franklin-Adams's lens the amount of movement required is about ·39 inch, or 3 per cent. of the whole separations. While in certain other combinations of separated simple lenses, such as the Huygenian eyepiece, and four lens-erecting eyepiece, the conditions of freedom from distortion and oblique achromatism are more or less independent of one another in these Cooke lenses ; provided that the two positive lenses are of similar but reversed shapes, the two conditions of rectilinearity and oblique achromatism are necessarily simultaneously attained.

The actual state of colour correction in these lenses as designed for stellar work is not such as to bring the actinic blue rays about G and the most luminous D rays to the same focus as in the case of camera lenses, but to bring the most highly actinic blue rays on the F side of G to the minimum, and therefore most intense focus, the most luminous yellow green and yellow rays being refracted to foci considerably beyond, and thus the most distinct image from a purely photographic point of view is obtained at the sacrifice of the visual distinctness. The two positive lenses are made of heavy barium silicate crown, having a refractive index of about 1·57 for the D ray, and the negative lens of flint glass having a refractive index of about 1·623 for the D ray.

Astigmatism and Flatness of Field.

3. The means whereby flatness of field combined with almost complete absence of astigmatism is attained really constitutes the most novel feature in the construction of these lenses.

If F = the principal focal length of any thin lens whatever, positive or negative, μ = its refractive index, and ϕ the angle of obliquity of any pencil of rays traversing the lens centrally, U the axial distance from the lens of any plane object placed perpendicular to the axis, and V the corresponding conjugate distance of the image from the lens, and y = the semi-aperture of the pencil when it traverses the lens, then

$$\frac{1}{V}=\frac{1}{F}-\frac{1}{U}+\frac{1}{8F^3}\omega y^2+\frac{\tan^2\phi}{2F}\cdot\frac{3\mu+1}{\mu}$$

wherein $\frac{1}{8F^3}\omega y^2$ = the spherical aberration to which all the pencils are subject and $\frac{\tan^2\phi}{2F}\cdot\frac{3\mu+1}{\mu}$ is the error of curvature of image as formed by the rays in the primary plane of the oblique pencil or in that plane containing the principal or central ray of the oblique pencil and the optic axis. Whereas the error of curvature for rays in the secondary plane of the same pencil or the plane at right angles to the former and also containing the principal ray is $\frac{\tan^2\phi}{2F}\cdot\frac{\mu+1}{\mu}$.

Thus the two curvature errors are

$$\frac{\tan^2\phi}{2F}\cdot\frac{3\mu+1}{\mu}\text{ and }\frac{\tan^2\phi}{2F}\cdot\frac{\mu+1}{\mu}$$

in the two planes, and, if $\mu = 1{\cdot}5$, are in the ratio 5·5 to 2·5 or 2·2 to 1, while their difference is $\frac{\tan^2\phi}{F}$ simply, which represents the astigmatic error of the lens. Thus the astigmatism depends solely upon the obliquity and the power of the lens and nothing else, while the two curvature errors depend also upon the refractive index, but are independent of U or V.

More than fifty years ago Petzval promulgated his now well-known theorem to the effect that if a system of thin positive and negative lenses combined in contact, and so arranged as to give a final image free from astigmatism away from the axis, then the image, if anastigmatic or free from astigmatism, will be curved to a radius whose reciprocal value is equal to $\Sigma\frac{1}{P\mu}-\Sigma\frac{1}{N.M}$

wherein $\Sigma\frac{1}{P}$ is the sum of the powers of the positive lenses and μ

their refractive index, and $\Sigma\frac{1}{N}$ is the sum of the powers of the negative lenses and M their refractive index. Supposing $\mu = M$; then obviously, if $\Sigma\frac{1}{P} = \Sigma\frac{1}{N}$, then the reciprocal value of the radius of curvature of the final anastigmatic image becomes zero, that is, the image is flat as well as free from astigmatism. But, unfortunately, as the focal powers of the negative lenses would in that case completely neutralise the focal powers of the positive lenses, there could be no real image. But if the refractive index for the positive lenses would be made higher than that for the negative lenses, which is contrary to the usual state of things, and in Petzval's day not feasible consistently with achromatism, then obviously the above formula equates to zero when $\Sigma\frac{1}{P} = \Sigma\frac{1}{N}\frac{\mu}{M}$ which leaves the powers of the positive lenses in excess, so that the combinations will yield an actual image. Petzval did not fully make known how he arrived at his theorem, nor did he state the conditions under which the oblique pencils of rays would have to traverse the lens system in order to form an anastigmatic image on a flat surface. As a matter of fact, if the Petzval condition is fulfilled in a system of thin lenses in contact having $\mu = M$ and having the diaphragm point at their centre, so that the oblique pencils all traverse the *centre* of the system, then the final curvature of image for rays in primary sections of the oblique pencils is given by $\Sigma\frac{1}{P}\frac{\tan^2\frac{\phi}{2}}{}\frac{3(\mu-M)}{\mu}$, and for rays in secondary planes by $\Sigma\frac{1}{P}\frac{\tan^2\frac{\phi}{2}}{}\cdot\frac{\mu-M}{\mu}$, that is, the two errors are in the ratio 3 to 1 instead of the usual 2.2 to 1 or so in the case of simple lenses. What is now further required in order to uncurve both these curved images and throw them back simultaneously on to the plane passing through the axial focus is that the lenses must be given certain shapes, and that the oblique pencils of light must be constrained to pass *eccentrically* through the system. This latter condition is secured by placing a circular aperture, considerably smaller than the lens aperture, at some distance either behind or in front of the system. The oblique pencils of rays are thus compelled to traverse the system eccentrically and then become subject to further corrections which I call diaphragm corrections. It can be proved that any diaphragm correction equal to + or −1 affecting the rays in secondary sections of oblique pencils is invariably accompanied by diaphragm corrections equal to + or −3 affecting rays in primary sections of the same pencils. Therefore by suitable diaphragm corrections introduced into any contact system of positive lenses of high refractive index combined with negative lenses of low refractive index fulfilling the formula $\Sigma\frac{1}{P\mu} - \Sigma\frac{1}{N}\frac{1}{M} = 0$ we can get the image formed by

secondary rays (of curvature = 1), and the image formed by primary rays (of curvature = 3) simultaneously thrown back upon the plane passing through the final axial focus, and thus obtain a positive image that is both flat and free from astigmatism.

Supposing we confine our attention to combinations of two simple thin positive lenses and one simple thin negative lens placed between and in contact in accordance with the above conditions we may go further than Petzval did, and consider what will happen, supposing the positive lenses are more and more separated from the negative lens. When in contact the power of the combinations on parallel rays is simply $\frac{1}{P_1}-\frac{1}{N}+\frac{1}{P_2}$; but it can be shown that when separated the power of the combinations becomes

$$\frac{1}{P_1}-\frac{1}{N}+\frac{1}{P_2}+\left\{\frac{S_1(P_2-N)+S_2(P_1-N)-S_1S_2}{P_1NP_2}\right\}$$

in which formula S_1 stands for the separation between first lens and negative lens, and S_2 the separation between the back positive lens and the negative lens.

The formula in brackets is the increment to the power due to separation alone.

Now while the power of a simple lens inevitably carries with it the proportional curvature errors

$$\frac{\tan^2\phi}{2F}\cdot\frac{3\mu+1}{\mu} \text{ and } \frac{\tan^2\phi}{2F}\cdot\frac{\mu+1}{\mu}$$

on the other hand it can be proved that the great increment to power brought about by separation in these triplets does *not* carry with it any curvature errors whatever, and is an unqualified gain. A contact triplet combination of the power 1 and aperture $\frac{F}{16}$ may, when separated, give a power equal to 2 or 3, with respective apertures of $\frac{F}{8}$ or $\frac{F}{5\frac{1}{3}}$, while its image will under certain conditions remain flat and free from astigmatism if the equation $\frac{1}{P_1\mu}-\frac{1}{N}\frac{1}{M}+\frac{1}{P_2\mu}=0$ is fulfilled. The next question that occurs is, supposing this equation, which we may call the Petzval condition, is not strictly fulfilled, either in that the negative lens is of a less power than the equation specifies, or that its refractive index is higher instead of lower than that of the positive lenses, as is the case in this Cooke lens for stellar photography; then can the image still be got sufficiently free from astigmatism if flat? The answer is in the affirmative. For although in the case of a contact combination giving a flat mean image, there is sure to be more or less considerable astig-

matism should the Petzval condition be departed from, yet since the resultant focal power is very largely increased by separation of the lenses, these errors are relatively decreased. If the focal power is doubled or trebled by separation, then the errors consequent upon not fulfilling the Petzval condition are relatively reduced to $\frac{1}{2}$ or $\frac{1}{3}$ respectively.

There are as often as not practical difficulties in the way of strictly following the Petzval condition, so that a method of construction which enables one, as it were, to drown the errors due to its non-fulfilment, and therefore becomes to a large extent independent of it, has much to recommend it. As in Mr. Franklin-Adams's type of lens the refractive index of the two positive lenses is 1·5853 for the $\tilde{G}$ ray, and that of the negative lens about 1·64615, then the Petzval condition demands that the power of the negative lens should be $\frac{1\cdot646}{1\cdot575}$, or 1·04 times the sum of the powers of the two positive lenses. As a matter of fact it is only about ·9475, a deviation of ·09, or 9 per cent. from the Petzval condition, which would be fatal in the case of a contact combination of lenses, but is of vanishing importance in this widely separated combination, 86 per cent. of whose equivalent focal power is due solely to separation, and whose residual astigmatism for the $\tilde{G}$ ray is thus reduced to an exceedingly small amount, namely, to one-seventh part.

For the *total* actual power of the lens $= \frac{1}{44\cdot6} = \cdot0224$, while the sum of the powers of the three lenses $=$

	$+\cdot039305$ for L_1
	$+\cdot058417$ for L_3
	$\cdot097722$
	$-\cdot094545$ for L_2
	$+\cdot003177$ total

Thus the actual power is seven times the simple sum of the powers of the constituent lenses, and therefore the astigmatic error as expressed by $\frac{1}{\mu P_1} - \frac{1}{MN} + \frac{1}{\mu P_2}$, wherein the Petzval condition is not fulfilled, is reduced or diluted seven times.

It is also obvious, since the power of the negative lens is so nearly equal to the sum of the powers of the positive lenses, that the mean curvature errors of the latter as expressed by $\frac{\tan^2\phi}{2}\left(\frac{1}{P_1}+\frac{1}{P_2}\right)\frac{2\mu+1}{\mu}$ are almost completely neutralised by the corresponding opposite errors of the negative lens or

$$\frac{\tan^2\phi}{2} \cdot \frac{1}{N} \cdot \frac{2M+1}{M}$$

Anyway, the remaining positive curvature errors are so small (in this case only about 5 per cent.) compared to the total curvature errors of either the positive lenses or negative lens that any diaphragm corrections left in the system for correcting these residual curvature errors are also merely residual. Hence the lens may be described as composed of three simple lenses each of which is so shaped as to be about free from diaphragm corrections when the rays first incident on the first lens have a certain assigned degree of divergence. In this particular case the lenses are symmetrically disposed to two conjugate focal planes, one being at a distance from the front lens L_1, equal to $1\frac{1}{2}$ times the distance of the conjugate focus from the back lens L_3. Also the first separation S_1 is about $1\frac{1}{2}$ times the second separation S_2, and the diaphragm point is supposed to be at the centre of the negative lens. As the oblique pencils pass centrally through the negative lens, no appreciable diaphragm corrections arise in its case, whatever its shape may be, but the two positive lenses have to be so shaped that the presence of the diaphragm a long way behind L_1 and in front of L_3, causing the oblique rays to traverse those lenses eccentrically, shall have a disturbing effect equal to -5 per cent. of the natural mean curvature errors of the two lenses as expressed by

$$\frac{\tan^2\phi}{2}\left(\frac{1}{P_1}+\frac{1}{P_2}\right)\frac{2\mu+1}{\mu}$$

In other words, the diaphragm corrections of the two lenses are simultaneously approximately eliminated or reduced to nearly zero. It is almost the same thing as saying that the two positive lenses are so shaped as to be about free from coma under the conditions in question. Thus, the diaphragm corrections being about eliminated from the system under the above conditions, it can be shown that the system as a whole will remain about free from diaphragm corrections should the plane object presented to L_1 on the left be removed by an infinite distance. For slight outward coma will then arise in L_1, giving rise to positive diaphragm corrections, but outward coma will simultaneously arise in L_3, giving rise to negative diaphragm corrections neutralising the former, so that the final image will retain its flatness, while at the same time inward coma arises in the negative lens, which neutralises the outward coma of the two positive lenses.

Also in order to retain the oblique achromatism of the lens under the new conditions, the negative lens should be moved nearer to L_1, as we have seen before when dealing with the chromatic correction and rectilinearity of the lens. The amount of this axial traverse of the negative lens required between the symmetrical case of copying or reducing 3 to 2 and the case of photographing infinitely distant objects varies roughly as the cube of the separation between the two

positive lenses, other things being equal, and in Mr. Franklin-Adams's 10-inch lens is actually about ·39 inch. The adjustments of a large lens of this type are necessarily of a very delicate nature, and means are provided both for tilting the negative lens in any direction with respect to the optic axis and also for moving it laterally in any direction parallel to itself. Perfect symmetry of field is thus more at command, and after that has been satisfactorily secured the negative lens is carefully fixed in position.

The actual photographs taken and exhibited by Mr. Franklin-Adams will sufficiently show the capabilities of the lens for stellar photography in cases where large regions of the sky are required to be depicted in one view.

1903 *July* 10.

Description of the Mount. By Alfred Taylor.

The mounting was designed to utilise two existing cameras with lenses of unequal sizes and focal lengths. Both cameras having to be in use at the same time, the results from one serving as a check on the other, perfect parallelism and sympathy of motion are of great importance. Rigidity in the mounting, with great ease of working, had therefore to be carefully kept in view in its construction.

The mounting is of the English type, there being no central pillar to interfere with a complete twelve hours' run for all positions of the telescope. This is an important factor where star photography is concerned. The polar axis consists of a cross-braced steel frame, square in section, the width at the middle being determined by a suitable spread of bearings to give length to the declination axis. Both ends of the polar axis run in double ball bearings, mounted in swivelling frames, which take the side thrust due to the inclination of the axis, while another ring of balls at the lower end takes the end thrust there. The swivelling frames much reduce the difficulty of adjusting the polar axis, and the risk of friction from error in alignment there would be with plain bearings. Ball bearings have been successfully adopted by T. Cooke & Sons for some time on equatorial mountings carrying telescopes anything over 6 inches in diameter, the reduction of friction and consequent ease of movement greatly conducing to regularity in running, the work required to be overcome by the driving clock being more uniform and considerably less than with plain bearings.

The hour-circle, tangent wheels for both axes, and declination circle are all of gun-metal.

The R.A. tangent wheel is 43 inches diameter, and has 1080 teeth. It is loose on its axis and can be clamped by a screw worked by bevel wheels and loose cords led conveniently near to the eye-ends of the guiding telescopes.

The hour-circle is 24 inches diameter, both edges divided on

silver to minutes of time, with fixed and movable verniers reading to two seconds of time.

It being Mr. Franklin-Adams's intention to do work in both hemispheres, the hour-circle and verniers have been figured to read both ways round, one set of the engraved figures being filled in with red wax for the southern hemisphere, and the other in black for the northern hemisphere.

The driving clock is by Messrs. Repsold, of Hamburg, and will run $2\frac{1}{2}$ hours without rewinding.

The spindle which communicates the motion of the clock to the mounting makes thirty revolutions in one minute. At the top end of this spindle a worm engages with a small wheel on the tangent shaft, so that between the clock and the large tangent wheel the number of driving wheels is kept small—a matter of some importance where accuracy of drive is concerned.

The slow motion consists of a train of mouse gear, driven by a small reversible electric motor, which can run up to 3,000 revolutions per minute. The slow motion is unlimited, *i.e.* the telescope can be run through a complete circle by means of the motor if desired.

The socket for the declination axis is of cast iron bushed at the ends with gun-metal and formed with large square flanges bolted to two sides of the polar axis frame. The length across the bearings is 3 feet.

The axis is also of cast iron and hollow. The end flanges carry the two cameras and brackets for the two guiding telescopes. These brackets have specially large flanges, which allow the guiders to be temporarily set 10° out of parallelism to view any corner of the rectangular field covered by the cameras. Each guider has a 6-inch achromatic object-glass of 6 feet focal length. The eye-end has a sliding stage allowing a rough circular movement of $\frac{5}{8}$ inch, and a sub-stage with a fine adjustment of $\frac{1}{10}$ inch, such as is used with microscopes.

The declination circle is fixed to the side of the polar axis frame. It is 2 feet 6 inches diameter, divided on silver to 5 minutes of arc, and reading by verniers (which turn with the cameras) to 10 seconds of arc. One of the verniers is read from alongside the eye-end of the adjoining guider by a telescope with a diagonal eyepiece, which can be turned in any convenient direction. The rim of the declination circle is divided and engraved with large figures for rough setting without using the reading microscope. The slow motion in declination is effected by means of a mouse feed gear, driven by a reversible electric motor—similar to that for the R.A. slow motion—driving a worm which works into a complete wheel. This gives an unlimited range to the slow motion and avoids the annoyance caused by coming to the end of the motion when the amount is limited. The driving wheel is 27 inch pitch diameter, and has 1,080 teeth. It is loose on the axis and may be clamped from the eye-end of the guider alongside.

Starting and reversing switches hang from near the eye-ends of both guiders, and the slow motions in both R.A. and declination are from thence under easy control.

The guiding telescopes have light and dark field electric illuminations to the cross wires, the leads for these and for the slow-motion motors passing up through the centre of the declination axis and down the guiders to the switches, the usual outside festoons of electric wires, and the consequent risk of their breakage, being thereby avoided.

The upper and lower end brackets for the polar axis will be supported on concrete, or stone, and brick piers.

The telescope will be erected in a wood shed 12 feet 6 inches wide × 20 feet 6 inches long, both inside measurement, resting on a brick foundation raised a little above the ground. The ridged roof is in two parts, each with four rollers, running on rails laid upon the upper side beams of the house. These beams are continued 10 feet beyond each end of the house ; the two halves of the roof run along these, and thus allow a clear overhead view for the instrument ; or by both halves being drawn to one end of the house, a clear view may be obtained at the other end down to the horizon.

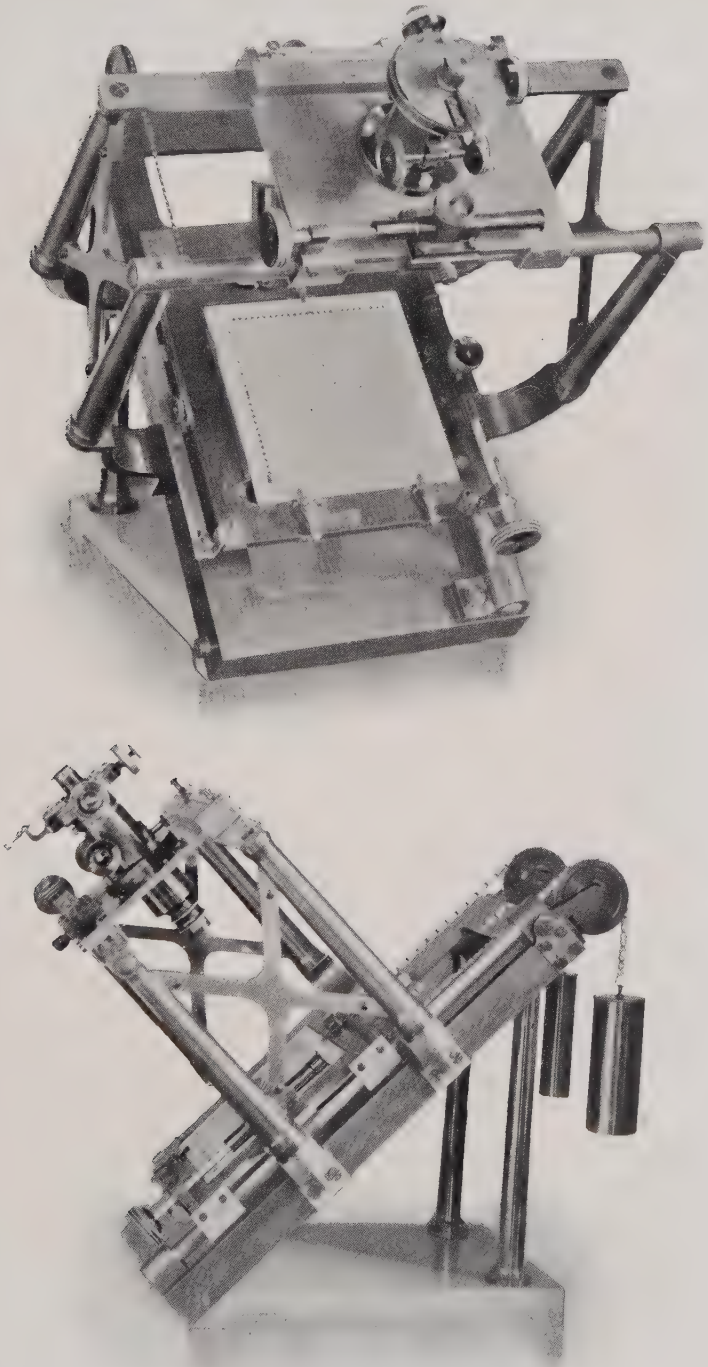
The sides and roof are covered with Willesden 4 ply waterproof paper.

The total weight of the mounting with cameras is about $2\frac{3}{4}$ tons, 100 lb. being the weight of the 10-in. lens.

On the New Greenwich Micrometer for measurement of Photographs of Eros.

(*Communicated by the Astronomer Royal.*)

The photographs of *Eros* taken for the determination of the solar parallax need to be measured with greater refinement than was necessary for the astrographic plates. A new micrometer was accordingly obtained from Messrs. Troughton and Simms early in 1903 in which the principle adopted by Mr. Hinks in the Cambridge micrometer (*Monthly Notices*, lxi. p. 444) of a divided glass scale in the focal plane moved by micrometer screws was employed. The new features in the Greenwich micrometer are (i) the microscope, which is arranged so that it can be used to give magnification 2 or 1, so as to adapt it to photographs taken either with the Astrographic telescope, focal length 11 ft. 3 in. (3·40 metres), or with the Thompson refractor, focal length 22 ft. 6 in. (6·80 metres) ; (ii) the glass scale is ruled with pairs of lines so spaced as to admit of very accurate pointing on the réseau line placed between them.



ROYAL OBSERVATORY, GREENWICH,

NEW MICROMETER FOR MEASUREMENT OF PHOTOGRAPHS OF EROS

The Microscope.

The microscope is designed so that a magnification of 1 or 2 by the object-glass can be used at will, the magnification unity being used for the photographs taken with the Thompson equatorial, and 2 for those taken with the Astrographic equatorial. In order that the field may be well covered it is desirable to have an object-glass of as long focus as practicable, the limit being determined by the condition that the eye-end should be at a height convenient for the measurer. An object-glass of 3 inches focus gives a distance of $13\frac{1}{2}$ inches from the plate to the focal plane, and this seemed suitable. Mr. Simms was able to supply an achromatic object-glass of 17 mm. diameter and 78 mm. focal length of the same make as one supplied to Sir David Gill for the Cape Observatory. With the magnification unity the object-glass would be at a distance of 156 mm. (roughly $6\frac{1}{4}$ inches) from the plate and at an equal distance from the focal plane. For magnification 2 the object-glass must be 39 mm. nearer the plate, and the focal plane is brought 39 mm. further from the plate. The object-glass is brought from the first to the second position by means of an adapter of the required length.

The eye-end of the microscope may be set in either of two alternative positions. This is arranged by a slot of the required length (39 mm.) which permits of motion in an outer tube. It is moved from one position to the other by hand and held in position by a ring clamp.

The adjustments for focus and runs are made (i) by a rack and pinion, which moves the whole microscope, and (ii) by a movement of the object-glass. The object-glass is carried by a sliding tube from which project two studs passing through slots in the outer tube. The studs are moved by antagonistic clamping rings working on the outer tube, and thus no rotation is given to the object-glass during its adjustment.

The general appearance of the micrometer is shown in the illustrations (Plate 15). The field of 20 mm. diameter which is required when magnification 2 is used makes it necessary that the tube of the microscope should have a large diameter, and this is an advantage in securing stability in the mounting.

As stated above the diameter of the object-glass is 17 mm., but to secure critical definition over the whole of the field a stop of 7 mm. diameter is placed inside the microscope 60 mm. from the object-glass. When the magnification unity is used the position of the stop is not changed, and is then 22 mm. from the object-glass. The stop is of such a size that at the extremities of the field the edge of the object-glass is just not reached, so that with magnification 2 the cone of rays always proceeds from a circular portion of the object-glass of 9·7 mm. diameter. With the magnification unity the stop being nearer,

a circle of 8·1 mm. of the object-glass is used. The following figures give the optical details of the microscope :

	mm.
Diameter of object-glass	17
Focal length	78
Diameter of stop	7

With magnification 1 the object-glass is 156 mm. from the plate and at an equal distance from the focal plane. With magnification 2 it is 117 mm. from the plate and 234 mm. from the focal plane.

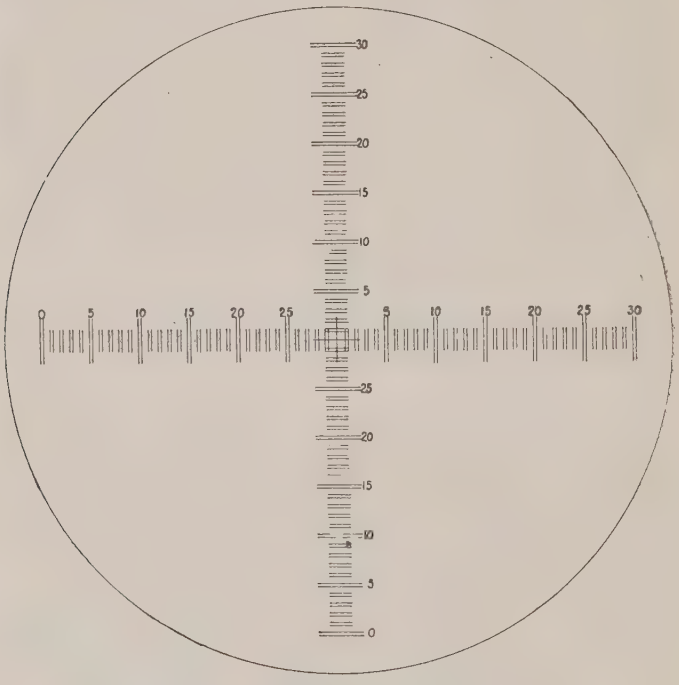


FIG. 1.—Arrangement of Scales on Glass Diaphragm.

The field to be measured extends 5 mm. from the centre on the plate, so that the image extends 5 mm. from the centre on each side for magnification 1 and 10 mm. for magnification 2.

The Diaphragm.

A glass diaphragm with divided scales in perpendicular directions is mounted on cross slides in the focal plane of the microscope, the slides being moved by micrometer screws. The

scales, which extend 10 mm. from the centre in the four directions, divide each 10 mm. into thirty equal parts by pairs of lines, the distance from the centre of one pair of lines to the next being $\frac{1}{3}$ mm., and equal to one revolution of either of the micrometer screws. In measuring the position of a star image within a réseau square the readings of the micrometer screws are first set at zero, and the image bisected on the central cross by means of the slow motions. The micrometer screws are then moved so that the réseau lines are bisected by the nearest pairs of lines of the scale. If there were no correction for runs the two réseau lines in each direction would be simultaneously bisected. In practice this is not the case, and the difference between the two micrometer readings supplies the correction for runs. The separation of each pair of lines is $\frac{1}{9}$ mm. or $\frac{1}{3}$ of the distance of the centre of each pair from the centre of the next. This separation is a very suitable one in relation to the breadth of a réseau line on a photograph. It also simplified the division of the scales, as, with the exception of the centre, 90 equidistant divisions were required with every third division omitted. The division of the diaphragm was made to agree exactly with the actual value of one revolution of the screw. For convenience of computation, therefore, the scale may be regarded as giving the integral part of the reading of the screw. With the magnification 2, used for the plates taken with the Astrographic telescope (scale 1 mm.$=1'$)

1 réseau interval$=300''=30$ divisions of the scale ;

so that in this case

1 rev. of screw$=1$ div. of scale$=10''$.

With magnification unity, used for the Thompson plates (scale 1 mm.$=30''$)

1 réseau interval$=150''=15$ divisions of the scale ;

so that again

1 rev. of screw$=1$ div. of scale$=10''$.

Micrometer Screws.

The total range through which the screws are used is one revolution. The value of one revolution was found sensibly equal to a mean division of the scale.

Division Errors of the Glass Diaphragm.

To determine the division errors of the diaphragm free from the optical distortion of the object-glass of the microscope the diaphragm was removed from the microscope and mounted in the

position of the photographic plate. A second similar diaphragm was substituted for it in the focal plane of the microscope. It was thus possible to compare different lengths of the diaphragm to be measured with a standard length at the centre of the field of the microscope. In this way the total length of the scale was subdivided into five lengths of twelve divisions each. These were further subdivided in a similar manner into four lengths of three divisions, and these again into single divisions.

The errors of the corresponding divisions on the left and right, or above and below the centre of the scale, are combined in the same way as the readings of the two réseau lines in forming the measured coordinate of a star's image. The weights thus given to the two errors in forming the mean are inversely proportional to the distance of the divisions from the centre of the scale. The following table gives the corrections for each division and the corrections to the mean for the two réseau lines :

Corrections for Division Errors of Glass Diaphragm, and the Correction to the Mean applicable when Magnification 2 is used.

Division.	*x* Coordinate. Left of Centre.	*x* Coordinate. Right of Centre.	*x* Correction to Mean.	*y* Coordinate. Below Centre.	*y* Coordinate. Above Centre.	*y* Correction to Mean.
	″	″	″	″	″	″
0	·000	−·031	−·031	·000	+·059	+·059
1	+·026	−·020	−·018	+·033	+·029	+·029
2	+·008	−·015	−·013	+·001	+·022	+·021
3	−·007	−·021	−·020	+·006	+·032	+·029
4	−·013	−·047	−·042	·000	+·005	+·004
5	+·003	−·032	−·026	+·015	+·018	+·016
6	+·009	−·006	−·003	+·030	+·044	+·041
7	+·047	+·049	+·049	+·070	+·084	+·080
8	+·048	+·032	+·036	+·072	+·078	+·076
9	+·036	+·018	+·023	+·060	+·053	+·055
10	+·012	+·016	+·015	+·037	+·051	+·046
11	+·001	+·006	+·004	+·026	+·053	+·043
12	+·010	+·005	+·007	+·048	+·049	+·049
13	+·038	+·015	+·025	+·060	+·065	+·063
14	+·008	+·009	+·009	+·040	+·051	+·046
15	−·011	−·007	−·009	+·012	+·040	+·026
16	−·082	−·079	−·081	−·052	−·041	−·047
17	−·083	−·103	−·092	−·061	−·062	−·061
18	−·119	−·106	−·114	−·081	−·068	−·076
19	−·109	−·096	−·104	−·072	−·046	−·062
20	−·081	−·113	−·092	−·052	−·051	−·052
21	−·093	−·121	−·101	−·058	−·059	−·058
22	−·107	−·126	−·112	−·081	−·052	−·073

Division.	*x* Coordinate. Left of Centre.	Right of Centre.	Correction to Mean.	*y* Coordinate. Below Centre.	Above Centre.	Correction to Mean.
	"	"	"	"	"	"
23	−·101	−·101	−·101	−·060	−·032	−·053
24	−·067	−·089	−·071	−·022	−·018	−·021
25	−·024	−·050	−·028	+·006	+·016	+·008
26	−·031	−·027	−·030	·000	+·030	+·004
27	−·029	−·023	−·028	+·005	+·029	+·007
28	−·033	−·055	−·034	−·014	−·004	−·013
29	−·040	−·069	−·041	+·003	−·019	+·002
30	−·031	·000	−·031	+·059	·000	+·059

With the magnification unity only the central half of the diaphragm is used. Thus if one réseau line is on division 5, to the right of the centre, the adjacent one falls on 20, to the left of the centre. The figuring to the right and top gives the integer set down in the measures. The corrections for division errors are formed by combining those given above, as in the following table :

Corrections for Division Errors of the Glass Diaphragm and Corrections to Mean applicable when Unit Magnification is used.

Divisions.	*x* Coordinate. Left of Centre.	Right of Centre.	Correction to Mean.	*y* Coordinate. Left of Centre.	Right of Centre.	Correction to Mean.
	"	"	"	"	"	"
15, 0	−·011	−·031	−·031	+·012	+·059	+·059
16, 1	−·082	−·020	−·024	−·052	+·029	+·024
2	−·083	−·015	−·024	−·061	+·022	+·011
3	−·119	−·021	−·041	−·081	+·032	+·009
4	−·109	−·047	−·064	−·072	+·005	−·016
5	−·081	−·032	−·048	−·052	+·018	−·005
6	−·093	−·006	−·041	−·058	+·044	+·003
7	−·107	+·049	−·024	−·081	+·084	+·007
8	−·101	+·032	−·039	−·060	+·078	+·004
9	−·067	+·018	−·033	−·022	+·053	+·008
10	−·024	+·016	−·011	+·006	+·051	+·021
11	−·031	+·006	−·021	·000	+·053	+·014
12	−·029	+·005	−·022	+·005	+·049	+·014
13	−·033	+·015	−·027	−·014	+·065	−·003
14	−·040	+·009	−·037	+·003	+·051	+·006
15	−·031	−·007	−·031	+·059	+·040	+·059

Distortion of the Field of the Object-glass.

The distortion of the field was determined for the magnification 2 by comparing intervals on the diaphragm (whose errors had

already been determined) with a standard interval. For this purpose the diaphragm was replaced in the focal plane of the microscope and the subsidiary scale mounted in the position of a photograph to be measured. The scale from end to end was divided into ten equal intervals, which were successively compared with the same interval of the subsidiary scale. There was some difficulty in the measures at the edge of the field owing to the lens being slightly over-corrected for flatness of field. In the following table are given the differences of each interval from the standard interval after correction for the errors of the scale. The heading gives the reading of the scale as measured from the centre :

	0^d–6^d	6^d–12^d	12^d–18^d	18^d–24^d	24^d–30^d
	″	″	″	″	″
From centre to right ...	+·077	+·002	·000	−·003	−·063
„ „ left ...	+ 004	+·038	+·051	−·008	−·093
„ „ top ...	+·054	+·050	+ 034	−·002	−·050
„ „ bottom ...	+·017	+·031	+·010	−·036	−·108
Mean	+·037	+·031	+·023	−·012	−·079

Applying a correction −″·035 per six divisions, which is equivalent to taking the scale at the centre, the correction for distortion is found to be :

0^d	6^d	12^d	18^d	24^d	30^d
″·000	+″·002	−″·002	−″·014	−″·061	−″·175

The reading 30^d corresponds to an angular distance of the object of $2\frac{1}{2}°$ from the centre of the object-glass. The above numbers indicate that the distortion is roughly proportional to the fifth power of the distance from the centre of the field. It is practically insensible up to 18^d, and increases rapidly after 24^d, or 240″ on the astrographic plate.

For the photographs taken with the Thompson equatorial the microscope is used with the magnification unity, and the field used can only extend to 15^d from the centre, within which range the distortion is quite insensible.

The amount of the distortion at different distances from the centre is shown in the diagram by the curve above the horizontal line. The *correction* for distortion, which is negative on the positive side of the centre, is shown by the dotted curve below the line. On the negative side the correction changes sign and is positive, being represented by the full-line curve. Practically the whole effect of the distortion appears in alteration of the "runs" of the micrometer, and the effect on the mean of the measures of a star image is very small, for in measuring the position of an image it is bisected at the centre of the field C, and in effect the distances from the two réseau lines between which it falls, *i.e.* CA and CB, are read. The error of runs is eliminated by combining these measures with weights in inverse

proportion to their distances from the centre. Measuring now in one direction, from right to left, the correction for distortion will be positive on one side of the centre and negative (indicated by the dotted curve) on the other. In the diagram the correction to the measure of the réseau line on the left is AM, to the réseau line on the right is zero, and the correction to the mean is CR.

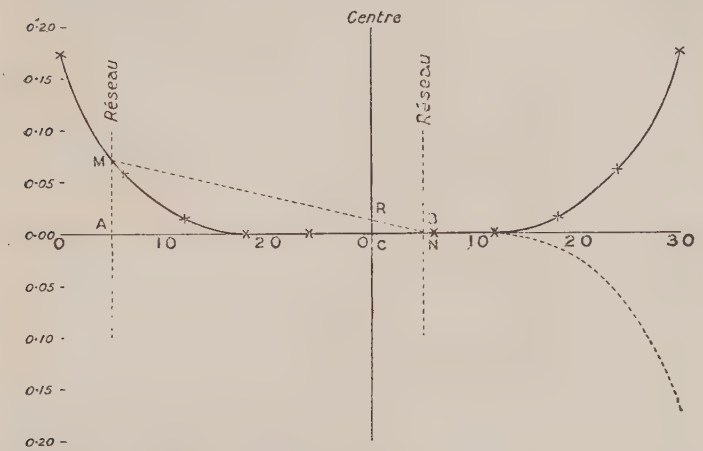


Fig. 2.—Distortion of Microscope Object-glass.

The position in the diagram corresponds to the maximum effect. The same result is shown in the following table:

Correction for Distortion.

Reading of Scale.	Left.	Right.	Mean.
d	"	"	"
0	+·175	·000	·000
6	+·061	+·002	+·012
12	+·014	−·002	+·003
18	+·002	−·014	−·003
24	−·002	−·061	−·012
30	·000	−·175	·000

Measurement of the Photographs of Eros.

On each photograph taken with the Astrographic equatorial measures are made of the planet, ten to twelve reference stars and six comparison stars. The reference stars are selected from M. Loewy's list within 50′ of the centre of the field, and, as far as practicable, symmetrically placed with respect to *Eros*. The comparison stars are approximately of the same brightness as *Eros*, within 25′ of the centre (so as to be well within the field of the Thompson plates) and symmetrically placed as regards *Eros*. On photographs with the Thompson equatorial the planet and

six comparison stars are measured. Four images are obtained on each photograph, and these are all measured, two by one observer and two by a second, in both direct and reversed positions of the plate. Four times as many measures are made of *Eros* as of the stars, each measurer measuring all four images of the planet in direct and reversed positions of the plate twice over, once before measuring the reference and comparison stars, and once after these measures. The scheme of measurement is as follows :

Measurer A. Images 1, 2, 3, 4 of planet, direct position of plate.
Images 1, 2 of stars, ,, ,, ,,
Images 1, 2, 3, 4 of planet, ,, ,, ,,
Images 1, 2, 3, 4 of planet, reversed position of plate.
Images 1, 2 of stars, ,, ,, ,,
Images 1, 2, 3, 4 of planet, ,, ,, ,,

Measurer B. Images 1, 2, 3, 4 of planet, direct position of plate.
Images 3, 4 of stars, ,, ,, ,,
Images 1, 2, 3, 4 of planet, ,, ,, ,,
Images 1, 2, 3, 4 of planet, reversed position of plate.
Images 3, 4 of stars, ,, ,, ,,
Images 1, 2, 3, 4 of planet, ,, ,, ,,

The large number of measures of *Eros* (thirty-two in all) makes them comparable with the total number of measures of reference and comparison stars. It is considered that the above system of measurement is amply sufficient to obtain the position of *Eros* with all the accuracy of which the photographs admit.

The progress made with the measurement of the photographs is as follows :

Of the 144 photographs with the Astrographic equatorial between 1900 October 1 and December 31, 121 have been completely measured, and ten by one measurer only.

Of the 102 photographs with the Thompson equatorial between the same dates fifty-five have been completely measured.

The stars used as comparison stars by micrometric observers have not yet been measured. This work will be taken in hand when the measures requisite for the reduction of the Greenwich photographs are completed.

The Accuracy of the Measures.

From measures made on a réseau and others made on specks on the film of a photograph it appears that the probable error of a pointing on a perfectly defined speck is $\pm''\cdot020$. This result is obtained with the magnification used in the actual measurement, and the probable error would doubtless be reduced if a higher power eyepiece were used.

For the determination of the accuracy with which the planet *Eros* is bisected, comparisons have been made between the two

pointings on *Eros* in the same position of the plate by the same measurer. As explained in the last paragraph these pointings were made after an interval of time during which other images were measured. The actual differences between Mr. Davidson's (C.D.) measures on plate 5306 are given as a specimen.

Plate 5306. *Measurer C.D.*

	Difference of 1st and 2nd Measures of Eros.	
	x	*y*
	"	"
Image 1 measured direct	− ·06	+ ·02
„ 2 „ „	− ·02	+ ·14
„ 3 „ „	+ ·08	+ ·04
„ 4 „ „	+ ·02	+ ·02
„ 1 „ reversed	+ ·01	− ·22
„ 2 „ „	− ·10	+ ·18
„ 3 „ „	+ ·23	+ ·04
„ 4 „ „	+ ·22	+ ·01
	± ·093	± ·084

Results of this nature, each depending on eight comparisons, are given below for a number of plates for the three measurers, Mr. Davidson (C.D.), Mr. Edney (D.E.), and Mr. Burkett (W.B.).

Plates taken with Astrographic Equatorial.

Mean Differences between Two Pointings on Eros by the same Measurer and in the same Position of the Plate.

	C.D.		D.E.		W.B.	
No.	*x*	*y*	*x*	*y*	*x*	*y*
	"	"	"	"	"	"
5275	± ·111	± ·073	± ·075	± ·118	...	...
5276	·080	·070	·133	·081	...	...
5278	·126	·064	·124	·133	...	...
5280	·121	·156	·131	·124	...	...
5283	·151	·183	...	...	± ·189	± ·183
5285	·131	·073	·100	·106	...	...
5286	·108	·132	...	...	·040	·090
5287	·064	·058	·103	·050	...	...
5288	...	...	·115	·080	·134	·071
5289	·101	·149	·078	·101	...	...
5290	·059	·150	...	...	·078	·146
5294	...	...	·113	·131	·090	·081
5297	·085	·184	...	...	·135	·119
5299	...	...	·120	·083	·155	·129

No.	C.D. x	C.D. y	D.E. x	D.E. y	W.B. x	W.B. y
	"	"	"	"	"	"
5300	·080	·173	·158	·079	...	...
5304	...	...	·070	·120	·098	·106
5306	·093	·084	·101	·166	...	...
5307	...	...	·164	·158	...	...
	± ·101	± ·120	± ·113	± ·109	± ·115	± ·116

Mean = ± ·110 ± ·115

PLATES TAKEN WITH THOMPSON EQUATORIAL.

Mean Differences between Two Pointings on Eros by the same Measurer and in the same Position of the Plate.

No.	C.D. x	C.D. y	D.E. x	D.E. y	W.B. x	W.B. y
	"	"	"	"	"	"
845	± ·086	± ·068	± ·069	± ·093	...	...
846	...	...	·101	·071	± ·094	± ·079
847	...	...	·105	·053	·123	·075
848	·069	·075	·096	·073	...	...
849	·048	·081	·081	·091	...	...
850	·051	·084	·069	·090	...	...
851	...	...	·079	·058	·100	·104
852	...	...	·081	·110	·038	·078
853	·068	·065	·071	·084	...	...
854	·140	·128	·065	·118	...	...
855	...	...	·108	·095	·065	·090
857	...	...	·086	·078	·140	·086
858	·064	·096	·085	·055	...	...
859	...	...	·084	·051	·054	·070
860	·132	·097	...	...	·067	·095
861	...	...	·103	·125	·066	·068
863	·063	·097	·070	·062	...	...
864	...	...	·070	·054	·064	·073
865	...	...	·073	·079	·111	·111
	± ·080	± ·088	± ·083	± ·080	± ·084	± ·084

Mean ± ·082 ± ·083

The above tables show that the accidental error of measurement is the same for the three measurers, and that it is considerably smaller for the Thompson than for the Astrographic photographs. The probable accidental error of a single pointing on *Eros* is

$$\pm''\cdot112 \times \frac{1}{\sqrt{2}} \times \cdot85 = \pm''\cdot067$$ for the Astrographic photographs

and

$$\pm''\cdot082 \times \frac{1}{\sqrt{2}} \times \cdot85 = \pm''\cdot049 \text{ for the Thompson photographs.}$$

The comparison of the results shows that the accidental errors of measurement are only a trifling part of the final error. No attempt has been made to analyse the sources of error in detail, though it seems clear that a considerable error is inherent in the star images.

In the following table the results obtained from different plates on October 26 and 27 with the two instruments are collected to show what accuracy may be expected in the results. These are the first two long nights as yet reduced, and may be taken as fairly representative nights. For the photographs with the Astrographic equatorial two results are given. In the first of these the position of *Eros* is obtained directly from twelve reference stars (within 50′ of the centre of the plate) taken from M. Loewy's Circular ; the second result is obtained from six comparison stars of about the same magnitude as *Eros*, whose positions are determined from the mean of the seven photographs. The same six comparison stars were used for the photographs with the Thompson Equatorial.

Residuals of Measured Places of Eros, 1900 *October* 26.

Astrographic Equatorial.

Plate No.	G.M.T.	Right Ascension. Parallax Factor.	R.A. T−O. Ref. stars.	R.A. T−O. Comp. stars.	R.A. Residuals. Ref. stars.	R.A. Residuals. Comp. stars.	Declination. Parallax Factor.	Dec. T−O. Ref. stars.	Dec. T−O. Comp. stars.	Dec. Residuals. Ref. stars.	Dec. Residuals. Comp. stars.
	h m s		s	s	s	s		″	″	″	″
5183	6 17 37	+1·44	+0·242	+0·227	+0·004	−0·011	−1·05	−1·28	−1·09	−0·02	+0·17
5184	7 0 1	+1·41	·240	·251	+ ·002	+ ·013	−0·83	1·53	1·32	− ·27	−0·06
5185	7 35 48	+1·34	·241	·224	+ ·003	− ·014	−0·66	1·33	1·16	− ·07	+ ·10
5186	11 25 5	+0·26	·244	·245	+ ·006	+ ·007	+0·04	1·07	1·22	+ ·19	+ ·04
5187	12 20 19	−0·09	·244	·242	+ ·006	+ ·004	+0·06	1·21	1·31	+ ·05	− ·05
5189	17 18 1	−1·41	·216	·230	− ·022	− ·008	−0·81	1·02	1·13	+ ·24	+ ·13
5190	17 39 31	−1·45	+ ·240	+ ·249	+ ·002	+ ·011	−0·97	−1·39	−1·63	− ·13	− ·37
Means	... ...		+ ·238	+ ·238	± ·0064	± ·0097		−1·26	−1·26		
Mean discordance for one plate			...		± ″·06	± ″·09				± ·14	± ·13
			Thompson Equatorial.								
787	6 43 14	+1·43	+ ·220	...	− ·011	...	−0·92	−1·23	...	+ ·01	...
788	7 5 42	+1·40	·238	...	+ ·007	...	−0·80	1·15	...	+ ·09	...
789	7 31 19	+1·35	·235	...	+ ·004	...	−0·68	1·27	...	− ·03	...
790	11 19 20	+0·30	·229	...	− ·002	...	+0·04	1·21	...	+ ·03	...
791	11 58 18	+0·05	·231	...	·000	...	+0·06	1·32	...	− ·08	...
792	17 6 21	−1·41	·238	...	+ ·007	...	−0·80	1·30	...	− ·06	...
793	17 27 15	−1·44	+ ·225	...	− ·006	...	−0·91	−1·21	...	+ ·03	...
Means	... ...		+ ·231	...	± ·0051	...		−1·24	...	± ·05	...
Mean discordance for one plate			...		± ″·05						

Astrographic Equatorial.

Plate No.	G.M.T.	Right Ascension: Parallax Factor.	R.A. T−O. Ref. stars.	R.A. T−O. Comp. stars.	R.A. Residuals. Ref. stars.	R.A. Residuals. Comp. stars.	Declination: Parallax Factor.	Dec. T−O. Ref. stars.	Dec. T−O. Comp. stars.	Dec. Residuals. Ref. stars.	Dec. Residuals. Comp. stars.
	h m s		s	s	s	s		″	″	″	″
5191	6 36 27	+1·44	+0·235	+0·231	+0·003	−0·001	−0·92	−1·31	−1·26	−0·13	−0·08
5192	7 2 1	+1·41	·227	·229	− ·005	− ·003	−0·80	1·37	1·30	− ·19	− ·12
5193	7 23 53	+1·37	·225	·216	− ·007	− ·016	−0·69	1·20	1·08	− ·02	+ ·10
5200	11 9 48	+0·32	·239	·240	+ ·007	+ ·008	+0·05	1·02	1·17	+ ·16	+ ·01
5201	11 30 28	+0·19	·240	·242	+ ·008	+ ·010	+0·06	1·14	1·17	+ ·04	+ ·01
5202	16 56 55	−1·42	·226	·224	− ·006	− ·008	−0·79	1·12	1·16	+ ·06	+ ·02
5203	17 25 1	−1·45	+ ·235	+ ·245	+ ·003	+ ·013	−0·93	−1·07	−1·10	+ ·11	+ ·08
Means			+ ·232	+ ·232	± ·0056	± ·0084		−1·18	−1·18		
Mean discordance for one plate ...					± ″·05	± ″·08				± ·10	± ·06

Thompson Equatorial.

794	7 18 1	+1 39	...	+0·234	...	−0·008	−0·71	...	−1·12	...	+0·04
795	7 49 30	+1·30	...	·232	...	− ·010	−0·56	...	−1·19	...	− ·03
796	11 37 52	+0·14	...	·248	...	+ ·006	+0·07	...	−1·12	...	+ ·04
797	12 15 6	−0·10	...	·248	...	+ ·006	+0·07	...	−1·17	...	− ·01
798	17 0 27	−1·42	...	·240	...	− ·002	−0·80	...	−1·19	...	− ·03
799	17 25 27	−1·46	...	·249	...	+ ·007	−0·93	...	−1·20	...	− ·04
Means			...	+ ·242	...	± ·0063		...	−1·16	...	
Mean discordance for one plate						± ″·06					± ·03

The mean discordances of the above observations are $\pm''\cdot089$ for the Astrographic plates and $\pm''\cdot048$ for the Thompson plates. The probable error of a position of *Eros* derived from an Astrographic plate is $\pm''\cdot089\times\cdot85\times\sqrt{\frac{7}{6}}=\pm''\cdot082$, and for a Thompson plate $\pm''\cdot048\times\cdot85\times\sqrt{\frac{6}{5}}=\pm''\cdot045$. The result for the Thompson photographs is entirely satisfactory. Some of the photographs with the Astrographic telescope on October 26 are not very good, and it seems likely that the probable error would be smaller than $\pm''\cdot082$ on good nights.. There is, however, an undoubted superiority of the Thompson photographs over those taken with the Astrographic equatorial.

As illustrating the accuracy which may be expected in the determination of the solar parallax from photographs of *Eros*, the following provisional results for October 26 and 27 may be of interest, but it is to be noted that they are not definitive, no correction for change of error in the ephemeris of *Eros* having been applied.

Instrument.	Mean Epochs G.M.T.		Deduced Solar Parallax.
	h m	h m	
Astrographic	Oct. 26 6 58	Oct. 26 17 29	8″783
,,	Oct. 26 17 29	Oct. 27 7 1	8·756
,,	Oct. 27 7 1	Oct. 27 17 11	8·771
Thompson	Oct. 26 7 7	Oct. 26 17 17	8·798
,,	Oct. 26 17 17	Oct. 27 7 34	8·805
,,	Oct. 27 7 34	Oct. 27 17 13	8·763

Royal Observatory, Greenwich:
1904 *May*.

Note on the Optical Distortion of the Microscope of one of the Oxford Machines for Measuring Astronomical Photographs. By H. C. Plummer, M.A.

The measures of the plates for the Oxford share of the Astrographic Chart Catalogue being now finished, a beginning has recently been made at the University Observatory in measuring with a modified form of instrument the photographs of the planet *Eros* which were taken in the autumn of 1900. The higher degree of accuracy which is demanded by parallax work has revealed defects in the optical arrangement of the apparatus which call for some examination. A brief note on the matter may be useful in drawing attention to a source of error which is possibly of general occurrence.

The type of measuring machine which has been in use at Oxford was described in *Monthly Notices*, vol. lv. p. 102. The modification which has been added recently consists in the pro-

vision of micrometer screws by means of which the graduated glass diaphragm in the eyepiece of the microscope can be moved parallel to the two perpendicular scales.

The objective of the microscope is a simple achromatic combination having an aperture of about 1·2 cm. and a focal length of nearly 5 cm. The distance between the plate and the scale is about 20 cm. and the magnification is 1·5, the réseau space being 0·5 cm. and 100 divisions of the scale being equal to 0·75 cm. The angular field which should be covered has thus a radius of $3\frac{1}{2}°$. It is clearly of great importance that there should be sharp definition without distortion over this field. The detection of the distortion which exists in the Oxford machine was due to the marked variation in the measured "runs," which was found on examination not to be due to irregularities in the réseau. For instance, a pair of réseau lines were set successively on the positions ·025 and ·525 on the scale. Several settings for "runs" were made in the two positions, and gave as a result −·0005 and +·0038 in the two cases, representing −0″·15 and +1″·14 respectively. The conditions under which these measures were obtained were identical except as regards the position of the réseau lines on the scale.

A partial calibration of the scale by comparing its divisions with distances measured by the micrometer screws was enough to show that neither the scales nor the screws could be held responsible for errors of this magnitude. The Zeiss scale leaves nothing to be desired in point of accuracy or clearness. The screws, on which the precision of the measures depends only in a minor degree, are also quite satisfactory. The experiments were therefore continued on the supposition that the discrepancies were due to optical distortion alone.

Systematic measures were made of the "runs" for two particular lines of a 5 mm. réseau at distances of ·05 along the X scale. The scale was first carefully centred, and measures were made along the whole length of the scale. Then the scale was shifted by turning the micrometer screws through five revolutions. In this way it was possible to prolong the measures beyond the range of the scale in its central position. The measured "runs" were then plotted as ordinates with abscissæ corresponding to the positions of the réseau lines on the scale. No distinction was made between equal distances from the centre to right or left, as the object was to find a distortion symmetrical about the centre. The points so obtained were smoothed by a graphical interpolation. The curve to which they approximated was parabolic in form, and corresponded fairly closely to the formula

$$E = 0^{1}\cdot 012\ (s - 0\cdot 5)^2 = 3''\cdot 6\, f^2,$$

where E is the effect of distortion on the "runs" and $s = f + 0\cdot 5$ is the scale reading of either end of the réseau line expressed as a fraction of the 100 scale-divisions which cover a réseau interval. Thus the effect, which vanishes when the scale is symmetrically

placed with respect to the two réseau lines, reaches the value $0^{r}\cdot0030 = 0''\cdot9$ at the extreme positions. If the effect is to be attributed to distortion symmetrical about the centre of the field, then that distortion must be represented by the formula

$$D = 0^{r}\cdot004\ s^3 = 1''\cdot2\ s^3,$$

The effect of the distortion is in the direction away from the centre of the field, so that the magnification increases in the outward direction.

It must be concluded then that distortion exists in the microscope now examined, and that it is of considerable amount. The effect of "runs" and distortion is that, instead of the true reading s, we obtain the reading*

$$r_1 = s + as + ds^3$$

for the preceding réseau line, where d is the distortion coefficient and a the value which the "runs" would have if there were no distortion. The reading for the following réseau line will be

$$r_2 = 1 - [1 - s + a(1-s) + d(1-s)^3],$$

whence the measured "runs" are found to be

$$r_1 - r_2 = a + d(1 - 3s + 3s^2).$$

The mean "runs" derived from the measures will therefore be $a + \frac{1}{2}d$, while the "runs" corresponding to $s = 0\cdot5$ will be $a + \frac{1}{4}d$. There are then three ways which suggest themselves of correcting the reading r_1: (a) by following the usual method and adopting the value $(1-s)r_1 + sr_2$; (b) by correcting r_1 for the mean "runs;" (c) by correcting r_1 for the "runs" corresponding to $s = 0\cdot5$. The effect of the distortion on the final measure will be in the three cases:

$$(a)\ \ ds(1-s)(2s-1);\ (b)\ \tfrac{1}{2}ds(2s^2-1);\ (c)\ \tfrac{1}{4}ds(4s^2-1).$$

These effects are here tabulated, the unit being $0^{r}\cdot0001 = 0''\cdot03$, with the value of d given above:

s	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
(a)	0	−3	−4	−3	−2	0	+2	+3	+ 4	+ 3	0
(b)	0	−2	−4	−5	−5	−5	−2	0	+ 4	+11	+20
(c)	0	−1	−2	−2	−1	0	+2	+7	+12	+20	+30

* Since symmetrical distortion alone is looked for, no term bs^2 is included. But such a term might be produced by a sensible tilt of the scale. This effect has been noticed in the case of the Cambridge machine, and is remedied by mechanical readjustment. It may be noticed that the effect of terms depending on even powers of s will be eliminated by measuring in direct and reversed positions of the plate, while the effect of radial distortion will not be so eliminated.

With the method (*a*), usually adopted for weighting the readings of the two réseau lines, the effect of distortion is much reduced, and its influence on the measures will scarcely be appreciable when these are only made to $0^{r}\cdot001$. But in more accurate work the defect is more serious. If a method like (*b*) or (*c*) be adopted, in which the reading of one réseau line alone is corrected for "runs," these being taken to be constant, it will always be possible to choose the nearer line to the measured object. Hence no attention need be paid to the large numbers occurring in the second half of the above table, and it is clear that method (*c*) is better than method (*b*), and even preferable, so far as the effect of distortion is concerned, to the ordinary method. In method (*c*), by adopting the "runs" as measured in the centre of the scale, we use the minimum measured "runs." Even so the measures are clearly over-corrected, and if we seek further for the most advantageous value to use over the range $s = 0$ to $s = 0\cdot5$ we arrive at the number $a + \frac{1}{8}d$, for this will make the mean error over the range zero. The outstanding error in the final coordinate is given here with the same units as in the above table :

s	0·0	0·1	0·2	0·3	0·4	0·5
Error	0·0	−0·5	−0·6	−0·4	+0·6	+2·5

The distortion arises from want of flatness in the field, and to this again is due a marked absence of good definition at a distance from the centre. Any appreciable distortion within the field to be covered, even when not complicated by bad definition, is inconvenient as requiring corrections which might be obviated by better optical means. That this can be done may be inferred from the performance of photographic lenses. It may be necessary to use a doublet, as in the Cambridge instrument, or at least a triple lens. It is to raise this question that the present note has been written. Can objectives be obtained which are sensibly free from distortion,* and is it necessary to substitute for the single objective an optical combination of greater complexity ?

Since the adoption of the réseau as an essential aid in the work of measuring, the machines employed for measuring astronomical photographs seem to fall under three distinct classes :

(*a*) The perpendicular threads in the focus of the microscope are fixed during the measurement. Relative motions in perpendicular directions between the plate and the microscope are produced by micrometer screws, so that the star and the four sides of the containing réseau square can be brought under the microscope threads. The mechanical arrangement of the apparatus

* The answer to this first question must be taken to be in the affirmative, for the Cambridge machine appears to be entirely free from distortion of an amount which can be detected. It is not so clear whether the capacity of a properly designed single objective is necessarily inadequate to the requirements.

must be of a high order of perfection, and the cost of the instrument must therefore be considerable. Accuracy will then depend on the quality of the screws, and will be affected by their deterioration with continued use. On the other hand little depends on the quality of the microscope. All that is required of it is good definition at the centre of the field.

(*b*) The microscope and plate are fixed during the measurement of the stars within a réseau square, and the threads in the focus of the microscope are moved by micrometer screws. The dependence on the quality of the screws is the same as in the former case, while the mechanical requirements are probably easier to satisfy. The centre of the réseau square is brought to the centre of the field of the microscope, and good definition without sensible distortion is required over an area corresponding to the square. If fixed threads, forming a square in the focus of the microscope, be added, we have the form of instrument adopted by Sir David Gill for the astrographic work at the Cape Observatory. By bringing the image of the réseau square into approximate coincidence with the fixed square a certain rapidity of measurement is attained, though at some sacrifice of accuracy owing to the partial neglect of "runs."

(*c*) The microscope is provided with two rectangular crossed scales, ruled on a glass plate and placed in the focal plane. The star to be measured is brought to the intersection of the scales, and the positions of the containing réseau lines on the scales must be determined. As this can be done with considerable accuracy by direct estimation, and a rigid connection between the plate and microscope while the measure is made is the only mechanical requirement, a useful instrument of this class can be made in the cheapest possible form (*British Association Report*, 1900, p. 676). Since a star may be anywhere within a réseau square, each arm of the crossed scale must have a length equal to a réseau space, and the microscope must give a field, free from distortion and of good definition, of a width equal to two réseau spaces.

The Oxford machine now examined belongs to this last type. The substitution of the graduated scale for the screw as the chief means of measurement is seen to increase the demand for optical efficiency in the microscope. Hence the importance of the question raised above as to the relative qualities of different types of objective. Perhaps it may be pointed out that the effective field required of the microscope may be reduced by extending the principle of the crossed scale and developing it into a reticle, *i.e.* a ruled réseau. If it were possible to rule such a reticle without sacrificing the clearness possessed by the simple scale the field might be reduced to little more than a single réseau space, while the rapidity of measuring all the stars within a single square might be increased.

Note on the Influence of the Plate Constants on the Accuracy of the Position of an Object measured on a Photograph. By H. C. Plummer, M.A.

When a preliminary solution is made for the constants of a plate on which the corrected positions of all the stars are required, as in the case of plates for the Astrographic Chart catalogue, the manner of proceeding is sufficiently obvious. It is then simply necessary to employ as large a number of comparison stars as possible, and to use the most accurate positions which can be found for them. The case is, however, rather different when the object is to find, not a solution which is to be applied over the whole extent of the plate, but one which is to be used to correct the measured coordinates of a particular object. This is what is required in parallax work, for example. Here questions arise which concern the choice and number of the comparison stars, and it is important from the point of view of economy to decide the relative degree of accuracy with which their coordinates should be measured. Mr. Filon has discussed the matter very thoroughly * and his main inferences are probably well known. The subject seems sufficiently important to justify discussion in a slightly different way, even if little that is new be added.

The equations from which the plate constants are calculated are of the form

$$ax+by+c=\Delta x\,;\ dx+ey+f=\Delta y$$

where Δx, Δy are the differences between the computed coordinates and the measured coordinates of the comparison stars, *i.e.* the projections on the axes of the distances Δr between the computed and the measured positions. It is assumed that the comparisons are of equal weight for both coordinates and for all the stars. The six-constant solution will be considered first. The normal equations are

$$\begin{aligned}
a[x^2]+b[xy]+c[x]&=[x\Delta x]\,;\ d[x^2]+e[xy]+f[x]=[x\Delta y]\\
a[xy]+b[y^2]+c[y]&=[y\Delta x]\,;\ d[xy]+e[y^2]+f[y]=[y\Delta y]\\
a[x]+b[y]+cn&=[\Delta x]\,;\ d[x]+e[y]+fn=[\Delta y]
\end{aligned}$$

where n is the number of comparison stars. But it is true, and formal proof might be given if necessary, that had the comparison been made according to any other axes the results would have been equivalent. Hence for theoretical purposes we may take as axes the principal axes of the momental ellipse at the centroid of particles of unit mass occupying the positions of the comparison

* *Monthly Notices*, vol. lxii. p. 561.

stars. The equations for the plate constants then take the simple form

$$\begin{aligned} a[x^2]&=[x\Delta x]\ ;\ d[x^2]=[x\Delta y]\\ b[y^2]&=[y\Delta x]\ ;\ e[y^2]=[y\Delta y]\\ cn\ &=[\Delta x]\ ;\ fn\ =[\Delta y]. \end{aligned}$$

If p is the probable error of a difference Δx or Δy, and p_a is the probable error of a, &c., then

$$\begin{aligned} &p_a{}^2[x^2]=p_b{}^2[y^2]=p_c{}^2n\\ &=p_d{}^2[x^2]=p_e{}^2[y^2]=p_f{}^2n=p^2 \end{aligned}$$

Now if x_o, y_o are the coordinates of the object whose position is required ; δx_o, δy_o the errors in x_o, y_o due to errors δa in a, &c. ; and p_x, p_y the probable errors of these coordinates so far as the effect of erroneous plate-constants is concerned,

$$\delta x_o=x_o\delta a+y_o\delta b+\delta c\ ;\ \delta y_o=x_o\delta d+y_o\delta e+\delta f.$$

Hence,

$$\begin{aligned} \delta x_o&=\Sigma\left\{\delta\Delta x\left(\frac{x_ox}{[x^2]}+\frac{y_oy}{[y^2]}+\frac{1}{n}\right)\right\}\\ \therefore\ p_x{}^2&=p^2\Sigma\left(\frac{x_ox}{[x^2]}+\frac{y_oy}{[y^2]}+\frac{1}{n}\right)^2\\ &=p^2\Sigma\left(\frac{x_o{}^2x^2}{[x^2]^2}+\frac{y_o{}^2y^2}{[y^2]^2}+\frac{1}{n^2}\right)\\ &=p^2\left(\frac{x_o{}^2}{[x^2]}+\frac{y_o{}^2}{[y^2]}+\frac{1}{n}\right)\ \ \ldots\quad\quad\ldots\quad\quad\ldots\quad(1)\\ &=p_y{}^2, \text{ similarly.} \end{aligned}$$

Again, if the star places be corrected for refraction and aberration, we have

$$e=a,\ d=-b.$$

Then a four-constant solution is possible and the normal equations are

$$\begin{aligned} a[x^2+y^2]&=[x\Delta x+y\Delta y]\ ;\ b[x^2+y^2]=[y\Delta x-x\Delta y]\\ cn&=[\Delta x]\ ;\qquad\qquad fn=[\Delta y] \end{aligned}$$

Hence

$$p_a{}^2[x^2+y^2]=p_b{}^2[x^2+y^2]=p_c{}^2n=p_f{}^2n=p^2$$

In this case

$$\begin{aligned} \delta x_o&=\Sigma\left\{\frac{x_o(x\delta\Delta x+y\delta\Delta y)}{[x^2+y^2]}+\frac{y_o(y\delta\Delta x-x\delta\Delta y)}{[x^2+y^2]}+\frac{\delta\Delta x}{n}\right\}\\ \therefore\ p_x{}^2&=p^2\Sigma\left\{\frac{x_ox+y_oy}{[x^2+y^2]}+\frac{1}{n}\right\}^2+p^2\Sigma\left\{\frac{x_oy-y_ox}{[x^2+y^2]}\right\}^2\\ &=p^2\Sigma\left\{\frac{x_o{}^2x^2+y_o{}^2y^2}{[x^2+y^2]^2}+\frac{1}{n}\right\}+p^2\Sigma\left\{\frac{x_o{}^2y^2+y_o{}^2x^2}{[x^2+y^2]^2}\right\}\\ &=p^2\left(\frac{x_o{}^2+y_o{}^2}{[x^2+y^2]}+\frac{1}{n}\right)\ \ \ldots\quad\quad\ldots\quad\quad\ldots\quad\quad\ldots\quad\quad\ldots\quad(2)\\ &=p_y{}^2, \text{ similarly.} \end{aligned}$$

The formulæ (1) and (2) have been obtained without making any assumption as to the configuration of the comparison stars. Moreover, by basing the calculation directly on the errors of the coordinates of the comparison stars, instead of using the probable errors of the plate constants, the necessity of assuming an absence of correlation between these constants has been avoided.

The most important inference is that the object required should be at, or very near, the origin—*i.e.* should coincide as closely as possible with the centroid of the comparison stars. If this condition be satisfied, the configuration of the stars and the method of reduction (by four or six constants) are practically immaterial. The probable errors, so far as they are due to erroneous constants, are given by

$$p_x = p_y = p/\sqrt{n}$$

If the object does not coincide with the centroid of the comparison stars it is desirable to make the moments of inertia about both axes in the plate large—that is, to have the stars widely scattered in both directions—if the six-constant solution is used ; but if the four-constant solution is used it is only necessary that one of the principal moments should be large. The four-constant solution is more valuable when the solution is to apply to the whole extent of the plate, particularly when the distribution of the comparison stars is bad.*

University Observatory, Oxford:
1904 *May* 12.

Note on the formulæ connecting "Standard Coordinates" with Right Ascension and Declination. By F. W. Dyson, M.A., F.R.S.

The simplest and most interesting formulæ from a mathematical point of view for determining Standard Coordinates are those given by M. Trépied in the Introduction to the Algiers Section of the *Astrographic Catalogue* (p. v), and all others are readily deduced from them.

* Mr. Filon, to whom I have had the advantage of showing the above note in MS., points out the desirability of stating definitely the assumptions which underlie the argument—namely, that "there is no correlation between the errors in the residuals of two given stars," and also that "there is no correlation between errors of measurement in the two coordinates" for each star. He adds that "distortion or ellipticity of the images may correlate the errors in x, y ; and, further, that if this distortion affect similarly all the star discs, it may introduce correlation into the x-measures of different stars." But in general, in work of the kind to which this note is intended to apply, the distortion of the images can be considered so slight that no serious effect of this nature need be apprehended.

Let x and y be the Standard Coordinates α, δ; the Right Ascension and Declination of a star; and A, D the Right Ascension and Declination of the centre of the photograph:

$$\left.\begin{aligned} \text{Then}\quad \sqrt{1+x^2+y^2}\,\sin\delta &= \sin D + y\cos D \\ \sqrt{1+x^2+y^2}\,\cos\delta\,\sin(\alpha-A) &= x \\ \sqrt{1+x^2+y^2}\,\cos\delta\,\cos(\alpha-A) &= \cos D - y\sin D \end{aligned}\right\} \begin{aligned} &\text{i} \\ &\text{ii} \\ &\text{iii} \end{aligned}$$

The second of these formulæ gives

$$x = \sin(\alpha-A)\cos\delta + \tfrac{1}{2}x(x^2+y^2) + \&c.$$

If $x = y = 1^\circ$

$$\tfrac{1}{2}x(x^2+y^2) = 1''\cdot09.$$

Thus the term $\frac{1}{2}x(x^2+y^2)$ may be readily tabulated as a small correction, or a diagram may be formed to give it. It is to be noted that $\frac{1}{2}x(x^2+y^2)$ is independent of the position of the plate's centre and very rough values of x and y are sufficient to give the correction accurately.

The first equation gives y in the terms of the declination. When $x = 0$ the equation becomes

$$\sqrt{1+y^2}\,\sin\delta = \sin D + y\cos D$$

and as $y = \tan(\delta-D)$ is known to be a solution of this equation, the following transformation of equation (i) is suggested.

$$\{y-\tan(\delta-D)\}\{y+\tan(\delta+D)\} = \frac{x^2\sin^2\delta}{\cos(\delta-D)\cos(\delta-D)},$$

$$\text{or}\quad y-\tan(\delta-D) = \frac{\frac{1}{2}x^2\tan\delta}{1+\frac{1}{2}\,\dfrac{y-\tan(\delta-D)}{\tan\delta}\cdot\dfrac{\cos(\delta-D)\cos(\delta+D)}{\cos^2\delta}}$$

For brevity put $y-\tan(\delta-D) = \frac{1}{2}x^2\tan\delta\,.\,z$

$$\text{and}\qquad \frac{\cos(\delta-D)\cos(\delta+D)}{\cos^2\delta} = \lambda^2$$

Then $z(1+\frac{1}{4}\lambda^2x^2\,.\,z) = 1$

$$\text{or}\qquad z = 2\,\frac{\sqrt{1+\lambda^2x^2}-1}{\lambda^2x^2}$$

$$= 1-\frac{\lambda^2x^2}{4}+\frac{1\cdot3}{4\cdot6}\lambda^4x^4\,\frac{1\cdot3\cdot5}{4\cdot6\cdot8}\lambda^6x^6+\&c.$$

Therefore

$$y = \tan(\delta - D) + \tfrac{1}{2}x^2 \tan\delta - \tfrac{1}{8}x^4 \tan\delta \frac{\cos(\delta-D)\cos(\delta+D)}{\cos^2\delta} + \tfrac{1}{16}x^6 \tan\delta \frac{\cos^2(\delta-D)\cos^2(\delta+D)}{\cos^2\delta} - \&c.$$

This formula shows explicitly the magnitude of the correction at different declinations to the approximate formula

$$y = \tan(\delta - D) + \tfrac{1}{2}x^2 \tan\delta.$$

For example, take the case where $\delta = 85°$, $D = 84°$, and $x = 1°$.

$$-\tfrac{1}{8}x^4 \tan\delta \frac{\cos(\delta-D)\cos(\delta+D)}{\cos^2\delta} = +\tfrac{1}{8}\frac{\sin^3 1° \cdot \cos 1° \cdot \cos 11°}{\sin^3 5°} \times 3600'' = 3''\cdot55.$$

For $\delta = 70°$ the correction can only amount to $0''\cdot06$.
This correction can be readily tabulated as far as $\delta = 85°$.

An Analysis of the Distribution of Stars on the 1,180 Plates in Zones +25° to +31° allotted to the University Observatory, Oxford, in connection with the International Astrographic Survey. By F. A. Bellamy.

1. The completion, on 1904 February 17, of the measurement in two positions of the 1,180 plates required for the zones +25° to +31°, which were allotted to the University Observatory, Oxford, in connection with the International Astrographic Catalogue of Stars to the eleventh magnitude, has afforded the means of revising the figures given by me in a paper (*Monthly Notices*, vol. lx. p. 12) concerning the distribution of stars. It is my intention now only to refer to that portion of the sky to which the resources of the observatory have been mainly devoted during the past twelve years, reserving consideration of the distribution of stars outside +25° to +31° to a future communication.

The plates discussed in the paper referred to were 513 in number, each with exposures of 6^m, 3^m, 20^s, and were exposed between 1892 January and 1899 June; other plates with exposures not precisely of these durations were omitted. In the present paper all plates are included, but only one of the three exposures has been dealt with throughout.

2. In selecting plates considered good, and rejecting others which did not show enough stars, constant use has been made of

the numbers shown on Argelander's charts for the areas covered by the plates. To obtain these numbers the outline of the area was ruled on glass and laid over the proper portion of the charts, when the number of stars included could be quickly counted. These numbers have been used to form Table II., which corresponds in all details with Table I. As a general rule a plate has always been accepted for measurement if it showed three times as many stars as Argelander. It was at once recognised that this rule would admit plates of inferior quality in the region of the Milky Way ; but this was accepted as an advantage from the point of view of limiting the work in these regions. Further, this rule was not adopted until a good deal of the early work had been done and many plates had been measured for which the Argelander ratio was less than 3·0. It may be advisable to repeat these plates, but it was determined to finish the whole work first, since a region not measured at all has stronger claims than one where measures only fall somewhat short of a definite standard. Among the 1,180 plates the Argelander ratio falls below 3·0 for 280 plates ; but for most of these it is greater than 2·5.

3. Table I. gives the total number of stars measured. Under R.A. 1^h, Dec. $+25°$ are given the total numbers on plates with centres at declination $+25°\ 0'$ and R.A.'s $1^h\ 0^m$, $1^h\ 8^m$, $1^h\ 16^m$, $1^h\ 24^m$, $1^h\ 32^m$, $1^h\ 40^m$, $1^h\ 48^m$, $1^h\ 56^m$; and since each plate in this zone includes about $4\frac{1}{2}$ min. each way from the centre this total will include about $4\frac{1}{2}^m$ of 0^h and about $\frac{1}{2}^m$ of 2^h ; in declination it contains the belt from $+23°\ 55'$ to $+26°\ 5'$. Hence all stars are included at least twice, and on the average about 2·6 times.

TABLE I.

Total Number of Stars Measured in each Hour and Zone.

Hours.	+25°.	+26°.	+27°.	+28°.	+29°.	+30°.	+31°.	Totals.	Stars measured to 1899 June.
0	2879 7	1936 8	1623 7	1582 7	1671 7	1233 7	1653 7	12577 50	9283 35
1	2214 8	1533 7	2440 8	1711 7	2097 6	1606 7	1813 6	13414 49	7227 28
2	1738 7	2052 8	1605 7	1261 6	2152 7	1544 6	1974 7	12326 48	5784 25
3	2051 8	1871 7	1864 8	2002 7	2793 7	1269 7	1308 7	13158 51	5428 21
4	1248 7	1330 8	445 7	1022 7	1388 6	1617 7	1245 6	8295 48	3224 18
5	3290 8	2890 7	2241 8	2392 6	1959 7	3977 6	5289 7	22038 49	6578 23
6	2975 7	4470 8	2823 7	4616 7	3450 7	4019 7	2991 7	25344 50	9322 20
7	3480 8	2005 7	1934 8	2630 7	2167 6	2931 7	2506 6	17653 49	6533 23
8	1828 7	1575 8	1420 7	2100 6	2643 7	1453 6	1367 7	12386 48	4751 22
9	1042 8	1323 7	1429 8	2908 7	1362 7	1526 7	1137 7	10727 51	4216 25
10	968 7	1557 8	1160 7	1630 7	994 6	1670 7	1290 6	9269 48	2733 19
11	696 8	967 7	1393 8	1366 6	1036 7	945 6	1210 7	7613 49	1263 12
12	565 7	1110 8	1189 7	2070 7	1065 7	1322 7	1295 7	8616 50	2407 18
13	617 8	622 7	882 8	1574 7	1064 6	1712 7	1318 6	7789 49	2574 23
14	742 7	1461 8	898 7	1059 6	1122 7	1499 6	1725 7	8506 48	2718 19
15	948 8	1042 7	1256 8	1401 7	1391 7	1992 7	1976 7	10006 51	1668 12
16	1256 7	2189 8	995 7	1845 7	1382 6	3449 7	1919 6	13035 48	2208 13
17	2084 8	6239 7	3020 8	1997 6	1735 7	4791 6	2942 7	22808 49	3024 12
18	2067 7	5185 8	5892 7	3459 7	2465 7	3891 7	4078 7	27037 50	5764 16

Hours.	+25°.	+26°.	+27°.	+28°.	+29°.	+30°.	+31°.	Totals.	Stars measured to 1899 June.
19	2747 8	4536 7	5937 8	5104 7	7687 6	8473 7	9127 6	43611 49	13189 17
20	5011 7	6805 8	5209 7	3034 6	4375 7	5517 6	7622 7	37573 48	12877 23
21	2857 8	2982 7	3229 8	2399 7	3138 7	5074 7	4373 7	24052 51	10673 27
22	2066 7	2829 8	2500 7	2423 7	1738 6	3143 7	3419 6	18118 48	9768 29
23	2059 8	1703 7	1857 8	1341 6	2398 7	2247 6	2249 7	13854 49	8327 23
Totals & Plates	47,428 180	60,212 180	53,241 180	52,926 160	53,272 160	66,900 160	65,826 160	399,805 1180	141,539 513

Table IV.

Oxford. Average Number of Stars measured on each Plate, and the Ratio of Oxford Numbers to Argelander.

Hour.	+25° Stars.	+25° Ratio.	+26° Stars.	+26° Ratio.	+27° Stars.	+27° Ratio.	+28° Stars.	+28° Ratio.	+29° Stars.	+29° Ratio.	+30° Stars.	+30° Ratio.	+31° Stars.	+31° Ratio.	Final Results. Stars.	Final Results. Ratio.	Final Results. Stars per Sq. Deg.	Previous Results. Stars.	Previous Results. Ratio.	Previous Results. Stars per Sq. Deg.
0	411	6·7	242	3·8	232	3·7	226	3·7	239	3·9	176	2·7	236	3·6	252	4·0	53·7	265	4·6	56
1	277	4·7	219	3·6	305	5·1	244	4·3	350	6·0	229	3·7	302	4·6	274	4·6	58·4	258	4·3	55
2	248	4·9	256	4·9	229	4·1	210	4·3	307	6·3	257	4·2	282	4·3	257	4·7	54·8	231	4 8	49
3	256	4·2	267	4·0	233	4·1	286	5·9	399	8·8	181	3·5	187	3·3	258	4·7	55·0	259	4·4	55
4	178	4·7	166	4·2	63	1·7	146	3·8	231	2·5	231	3·6	208	3·5	173	3·3	36·7	179	4·4	38
5	411	3·8	413	3·9	280	3·0	399	4·5	280	3·1	663	5·6	756	6·2	450	4·3	96·0	286	3·5	61
6	425	2·9	559	3·7	403	3·1	659	5·9	493	4·7	574	4·6	427	3·8	507	4·0	108·1	466	3·7	99
7	435	4·7	287	3·0	242	2·8	376	4·6	361	4·9	419	4·9	418	4·7	360	4·2	76·8	284	3·3	60

Hour.	+25° Stars.	Ratio.	+26° Stars.	Ratio.	+27° Stars.	Ratio.	+28° Stars.	Ratio.	+29° Stars.	Ratio.	+30° Stars.	Ratio.	+31° Stars.	Ratio.	Final Results. Stars.	Ratio.	Stars per Sq. Deg.	Previous Results. Stars.	Ratio.	Stars per Sq. Deg.
8	261	4·1	197	2·9	203	3·2	350	6·5	378	7·2	242	3·7	195	2·7	258	4·1	55·0	216	3·4	46
9	130	2·4	189	3·3	180	3·4	415	8·6	195	4·3	218	4·0	162	2·7	210	4·0	44·8	169	3·3	36
10	138	3·0	194	4·0	166	3·5	233	5·3	166	4·1	239	4·9	215	4·0	193	4·1	41·2	144	3·1	31
11	87	2·2	138	3·4	174	4·3	228	5·6	148	4·1	158	3·5	173	4·2	155	3·9	33·1	105	2·6	22
12	81	1·8	139	3·1	170	3·9	296	7·8	152	4·3	189	4·0	188	3·8	172	4·0	36·7	134	3·2	29
13	77	2·0	89	2·1	110	3·1	225	5·9	177	4·8	245	5·3	220	4·9	159	4·0	33·9	112	2·9	24
14	106	2·3	183	3·9	128	2·7	176	4·4	160	4·4	250	5·1	246	5·4	177	4·0	37·7	143	3·4	30
15	119	2·5	149	2·9	157	3 4	200	4·6	199	4·5	270	5·5	282	5·7	196	4·1	41·8	139	2·9	30
16	179	3·1	274	4·8	142	2·4	263	4·8	230	4·5	493	7·8	320	5·0	272	4·7	58·0	170	3·0	36
17	261	3·4	891	11·8	378	5·3	333	4·4	248	3·2	799	9·6	420	6·0	465	6·2	99·4	252	3·3	54
18	295	3·0	648	6·5	842	9·1	494	5·4	352	3·9	556	5·4	583	5·5	541	5·5	115·4	360	3·7	77
19	343	2·8	648	5·2	742	5·9	729	5·6	1281	5·2	1210	7·4	1521	8·8	890	5·9	189·9	776	6·3	165
20	716	6·2	851	7·2	744	6·2	506	4·2	625	4·8	920	5·9	1089	6·8	783	6·0	166·9	560	4·7	119
21	357	4·4	426	5·0	404	4·1	343	3·4	448	4·7	725	6·5	625	5·7	472	4·9	100·6	395	4·4	84
22	295	4·0	354	4·9	357	4·4	346	4·4	290	3·5	449	4·6	570	5·9	377	4·6	80·4	337	4·4	72
23	257	4·0	243	3·6	232	3·4	224	3·4	343	5·3	375	4·7	321	4·1	283	4·1	60·3	252	4·0	54
Average	263·5		334·5		295·8		330·8		333·0		418·1		411·4		338 8		72·3	276		58·7
Plates	180		180		180		160		160		160		160		1180			513		
Ratio ...		3·74		4·64		4·24		4·96		4·63		5·30		5·18		4·66			4·0	

TABLE II.

Total Number of B.D. Stars included in the same Areas as in Table I.

Hours.	+25°.	+26°.	+27°.	+28°.	+29°.	+30.	+31°.	Totals.
0	431	512	443	429	430	461	462	3168
1	467	420	482	403	349	437	392	2950
2	352	419	391	292	340	365	461	2620
3	493	465	451	339	316	358	390	2812
4	267	315	268	269	563	452	359	2493
5	872	748	739	534	627	711	850	5081
6	1019	1201	897	776	741	865	781	6280
7	740	666	689	570	446	596	531	4238
8	448	545	443	323	368	389	503	3019
9	434	396	420	338	318	383	413	2702
10	326	388	329	306	245	339	324	2257
11	321	287	321	242	254	269	290	1982
12	306	356	305	265	248	328	341	2150
13	311	289	287	265	224	324	267	1967
14	327	378	327	242	256	293	322	2145
15	384	359	368	302	308	364	348	2433
16	401	457	413	385	304	440	381	2781
17	618	527	573	450	549	498	494	3709
18	688	803	648	639	638	722	747	4885
19	972	869	998	908	1468	1141	1036	7392
20	806	941	839	729	910	942	1114	6281
21	653	597	794	706	662	778	773	4963
22	514	575	572	555	494	685	575	3970
23	517	470	544	397	446	473	548	3395
Total ...	12,667	12,983	12,542	10,664	11,502	12,613	12,702	85,673

4. Features worthy of remark are the fairly uniform totals, about 2,000 and 3,000, for the hours outside the Milky Way, and the similarity of the average of the three hours densest in stars in the two parts of the Milky Way which cut the Oxford zones about 6^h and 19^h—namely, 5,200 and 6,200. In the Oxford results the numbers are between 8,000 and 13,000 for similar hours in the former case, but in the Milky Way (5^h to 8^h, and 18^h to 21^h) the average Oxford numbers are 21,700 and 36,100; and, as I shall show later in this paper, the average for the 6^h portion might be increased to 23,900, while the 19^h portion would become 47,100. For Argelander the increase from 6^h to 19^h (each mean of three hours) is 19 per cent.; for the Oxford measures the increase is as much as 66 per cent., and might readily be increased to more than 100 per cent. without increasing

the exposure of 3 min. I have always had a difficulty in obtaining a plate that showed 1,000 stars with a 3 min. exposure in the 6^h part of the Milky Way, but have numerous plates containing 2,000, 3,000, 4,000, and one nearly 5,000 stars with this exposure in the 19^h part.

This is an independent proof of the statement that has been made more than once that Argelander did not observe all the 9·5 mag. stars in the Milky Way, owing to the enormous number that had to be dealt with, but unconsciously decreased his scale by 0·2 mag. The difference between the Oxford and Argelander percentages would be much less if he had added a thousand stars in these two tenths of a magnitude; even a tenth of a magnitude fainter would have added a large number of stars where the increase in number is so rapid.

5. Now this seems to have a very important bearing on Kapteyn's conclusion, which is also supported by Professor S. Newcomb (*Astronomical Journal*, vol. xxi. p. 155), that "the stars of the Milky Way are in general bluer than the stars in other regions of the sky" (*C.P.D.* vol. i. p. [22]). For here the comparison is not between the Milky Way and other regions, but simply between two parts of the Milky Way. If the effect is to be attributed to a physical characteristic of the stars, then we must further conclude that there are variations of stellar luminosity in different parts of the Milky Way itself, in addition to the broad distinction between the Galaxy and other parts of the sky. It is surely simpler and more natural to infer a want of uniformity in the *Bonner Durchmusterung*, and thereby to explain the facts on the single hypothesis that Argelander's limiting magnitude in each region of the sky is a function of the star density.

6. Table III. exhibits all the instances when over 1,000 stars occur on those plates adopted for measurement.

Table III.

Dec.	R.A.	No. of Stars in 3 Min.	Plate.	Dec.	R.A.	No. of Stars in 3 Min.	Plate.
°	h m			°	h m		
+26	5 44	1158	1169		17 51	1489	1958
	5 52	1456	1170	+26	18 40	1267	1086
	7 28	1020	1536	+27	18 44	1146	1107
+28	6 18	1287	1734	+28	18 9	1489	1567
+31	5 37	1079	2298	+28	18 36	1165	1560
					18 45	1574	1561
				+29	18 22	1396	1578
+26	17 36	1223	1078		18 40	1401	1579
	17 44	1436	1079	+30	18 27	1064	1595
	17 52	1058	1080		18 36	1042	1596
+30	17 42	1224	1957		18 45	1084	1597

Dec.	R.A.	No. of Stars in 3 Min.	Plate.	Dec.	R.A.	No. of Stars in 3 Min.	Plate.
°	h m			°	h m		
+26	19 36	1763	1117		19 43	2210	2008
	19 52	1428	1119		19 52	2118	2001
+27	19 8	1398	1109	+25	20 12	1181	2289
	19 16	1230	1103	+26	20 0	1808	1120
	19 48	1176	1112		20 8	1774	1121
+28	19 12	1027	1223		20 16	1549	1212
	19 21	1100	474	+30	20 6	2097	1568
	19 48	1158	858		20 15	1496	1569
+29	19 7	2723	1562		20 24	2310	1564
	19 16	2856	1563		20 33	1541	1570
+29	19 34	1996	851		20 42	1823	1571
	19 43	2440	852		20 51	2703	1565
	19 52	1270	853	+31	20 1	1341	2016
+30	19 12	1057	1876		20 10	1228	1835
	19 39	4920	1582		20 19	1325	1834
	19 48	4097	1583		20 28	1310	1836
	19 57	2561	1584		20 46	1017	2015
+31	19 7	1178	1605	+30	21 9	1680	1566
	19 16	1342	1999		21 18	1802	1585
	19 25	1923	2000				
	19 34	2180	1606				

From this it can be seen that whilst we have in 5^h, 6^h, and 7^h only five plates with over 1,000 in 3 min. exposure we have fifty-five plates in the later part of the Milky Way, where it is more expanded, with more than 1,000 stars; of these twelve have more than 2,000. The greatest number is obtained on plate 1582, with the centre at R.A. $19^h\ 39^m$ +30°.

7. Taking the numbers in Table I. and dividing them by the subscript figures, the average numbers of stars for each hour, each zone, and all zones combined are obtained, and are given in Table IV. (see p. 652) on the left-hand side of each zone; when the numbers in Table I. are divided by the corresponding numbers in Table II. the ratios for Oxford to Argelander are found, and these form the quantities given on the right-hand side of the different zones.

8. Though the ratio of three times Argelander is well exceeded, except in 3^h, in the average for each hour of the seven zones combined and for each zone of twenty-four hours, some hours of all the zones, except +28°, show that this minimum has not been maintained; this is particularly the case in zone +25°, in the hours where there are least stars per square degree. This zone was that first photographed—mostly eleven

years ago—when the plates used were less sensitive, and a number of plates had been measured previous to the adoption of the standard, as mentioned in the beginning of this paper. A considerable portion of +26° and +27° had been re-photographed before the measurement of the plates had advanced very far; this especially applied to the latter part of the Milky Way in +26°. In the case of some of the hours in +31° the falling off in numbers was largely due to the quality of the plates supplied; though many were retaken, too much time had been spent over the first plates of the region to justify the whole being rejected.

9. The average given at the foot of Table IV. (see p. 652) indicates the improvement in the sensitiveness of the plates. The most southern zone was first photographed; the most northern zones were only photographed within the last two or three years.

The Argelander ratio for the whole 1,180 plates is 4·7; and the average number of stars measured on a plate is 339.

10. So far, except § 6, we have dealt with *actual measures* executed at Oxford. But it has been remarked (see § 2) that there is some lack of uniformity of procedure. The main object kept in view was to measure a fair number of stars all round the sky, and not necessarily to reach the same limit of magnitude in all R.A.'s. To do this we should have to measure an enormous number of stars in the Milky Way, which would have extended the work unduly. It is, however, a simple matter to *count* these plates, which gives all the information required for the study of stellar distribution; and for such counts I proceed to determine the corrections to the above tables necessary to represent the same limits of stellar magnitude round the sky. I will first state exactly how the work was limited in the rich regions.

11. As a first experiment only those stars showing all three images (6^m, 3^m, 20^s) were measured (ten plates only), when in this way at least three times Argelander's number could be obtained. This was found in practice to be too severe a limitation, and the 20 sec. exposure was accordingly increased to 60 sec., which was generally found sufficient to give an Argelander ratio exceeding 3·0; forty-five plates—mostly in the Milky Way—were so measured, *i.e.* those photographed in one minute. In addition to these about eighty-three plates had exposures differing slightly from the regulation 6^m 3^m 20^s; all the remaining 1,042 plates received these exposures.

Table V.

Zone. Centre. +	R.A.	Plate.	Number Measured.	3^m Images that might have been Measured.	Additional Stars.
°	h m				
29	2 55	1860	†117	468	351
25	3 40	1266	†221	422	201
28	3 45	1317	†297	705	408

Zone. +	Centre. R.A. h m	Plate.	Number Measured.	3m Images that might have been Measured.	Additional Stars.
°					
26	5 44	1169	†687	1158	471
26	5 52	1170	†538	1456	918
29	5 28	1309	†252	628	376
29	5 55	1311	†261	729	468
28	6 0	1733	†329	1875	1546
26	6 8	1171	†455	989	534
28	6 45	1497	†358	882	524
29	6 31	1314	†242	592	350
29	6 58	1315	†312	657	345
26	7 28	1536	†368	1020	652
28	7 3	1499	†324	836	512
26	12 48	1186	†164	276	112
27	17 40	1095	†197	800	603
30	17 51	1958	†345	1489	1144
25	18 4	1198	†410	870	460
25	18 28	1199	†443	943	500
25	18 44	1200	†290	916	626
28	18 9	1567	†463	1375	912
28	18 36	1560	†551	1165	614
28	18 45	1561	†686	1574	888
29	18 22	1578	†611	1396	785
29	18 40	1579	†449	1401	952
30	18 27	1595	†554	1064	510
30	18 36	1596	†496	1042	546
30	18 45	1597	†582	1084	502
27	19 16	1103	†273	1230	957
26	19 36	1117	†292	1763	1471
26	19 44	1118	†203	702	499
26	19 52	1119	†387	1428	1041
28	19 3	1222	†357	974	617
28	19 12	1223	†506	1027	521
29	19 7	1562	†1133	2723	1590
29	19 16	1653	†1097	2856	1759
29	19 52	853	†365	1270	905
30	19 39	1582	†2102	4920	2818
30	19 48	1583	†1469	4097	2628
30	19 57	1584	†1120	2561	1441
31	19 7	1605	†591	1178	587

Zone. +	Centre. R.A. h m	Plate.	Number Measured.	3^m Images that might have been Measured.	Additional Stars.
31	19 34	1606	†943	2180	1237
26	20 16	1212	†683	1549	866
27	20 20	1225	†395	802	407
30	20 6	1568	†916	2097	1181
30	20 15	1569	†852	1496	644
30	20 24	1564	†909	2310	1401
30	20 33	1570	†754	1541	787
30	20 42	1571	†872	1823	951
30	20 51	1565	†1214	2703	1489
27	21 32	1230	†236	615	379
28	21 9	1240	†242	524	282
28	21 27	1215	†272	738	466
30	21 18	1585	†792	1802	1010
29	22 34	1466	†229	550	321

12. Table V. contains the data relating to the 45 plates with 6^m, 3^m, 1^m exposures, and the 10 plates with 6^m, 3^m, 20^s, upon which only those stars showing all three exposures were measured. The number actually measured is preceded by this symbol, † ; the number that would have been measured in the ordinary way is given in the next column ; the difference or additional stars—those showing the 3^m, but not the 20^s or 1^m exposure—that might be measured at any time are in the last column. If these extra stars be incorporated in the *measured* totals given in Tables I. and IV. we should obtain the amended results exhibited in Table VI., with a total addition of 44,462 stars.

13. To measure these stars it would require nearly 90,000 bisections in each coordinate to be made. This would be an additional year's work for comparatively little advantage, for it will be noticed that nearly all these 44,000 stars are within the region covered by the Milky Way, where, for star *positions*, they can be very well spared, as most of them are probably fainter than the 10·5 magnitude ; but when considering the distribution of stars they should not be omitted in discussing the results. In the latter case any discussion of photographic results is rendered uncertain by that extremely variable quantity the climatic conditions. A small variation of exposure is almost insignificant compared to those conditions and to the varying chemical properties in the emulsion from the time the plate is coated till the finished negative is obtained.

TABLE VI.

Difference between the Stars Measured that show the Three Images and those Counted for the 3m Exposures.

Hour.	Sums for Hours and Zones. 25°.	26°.	27°.	28°.	29°.	30°.	31°.	Additional Stars *Counted*, 3m Exposure.	Stars *Measured* added to previous Column.	Ratio.
0	...	...	...	...	...	...	...	0	12577	3·97
1	...	...	...	...	...	...	...	0	13414	4·55
2	...	...	...	...	351	...	...	351	12677	4·84
3	201	...	...	408	...	...	...	609	13767	4·91
4	...	...	...	...	...	...	...	0	8295	3·33
5	...	1389	...	...	844	...	...	2233	24271	4·78
6	...	534	...	2070	695	...	...	3299	28643	4·56
7	...	652	...	512	...	...	...	1164	18817	4·44
8	...	...	...	...	...	...	...	0	12386	4·11
9	...	...	...	...	...	...	...	0	10727	3·96
10	...	...	...	...	...	...	...	0	9269	4·11
11	...	...	...	...	...	...	...	0	7613	3·85
12	...	112	...	...	...	...	...	112	8728	4·06
13	...	...	...	...	...	...	...	0	7789	3·96
14	...	...	...	...	...	...	...	0	8506	3·97
15	...	...	...	...	...	...	...	0	10006	4·14
16	...	...	...	...	...	...	...	0	13035	4·69
17	...	...	...	...	...	1144	...	1144	23952	6·46
18	1586	...	...	2414	1737	1558	...	7295	34332	7·03
19	...	3011	957	1138	4254	6887	1824	18071	61682	8·34
20	...	866	407	...	...	6453	...	7726	45299	7·21
21	...	...	379	748	...	1010	...	2137	26189	5·28
22	...	...	...	...	321	...	...	321	18439	4·64
23	...	...	...	...	...	...	...	0	13854	4·09
Sums	1787	6564	1743	7290	8202	17052	1824	44462	444267	5·19 Mean

14. A comparison of the final results with those upon which the previous paper communicated to the Society (*Monthly Notices*, vol. lx. p. 12) in 1899 were based may be of interest. It will be noticed (Table VII.) that while more than twice as many plates have been used in the final results nearly three times the number of stars have been included. The number of stars (those showing both the 6m and 3m images) per square degree of ranges from 56·2 in +25° zone to 89·2 in +30° zone; if the extra stars had been measured the range would be 57·8 to 111·8. The value for the whole of the seven zones photographed at Oxford being 72·3.

for the stars measured, this is very similar to that found (71·2) at the Royal Observatory, Greenwich, for zones +65° to +69° (inclusive).

15. The last six columns in Table IV. give the details for the separate hours in a similar manner to the results given in Table VII. for each whole zone; the earlier results are added for comparison. In plotting these new values on plate 1 of the former paper I find almost precisely the same *form* of curve results, but generally at a higher level. It may be remarked that the maximum number of stars measured per square degree is 189·9 at 19^h (R.A. $19^h\ 0^m$ to $20^h\ 0^m$), and taking only those numbers that are above the mean (72·3) the Milky Way may be considered to extend from R.A. $16\frac{1}{4}^h$ to $22\frac{1}{2}^h$; the width about the 6^h portion is very much narrower, and, according to the Oxford survey, of about half the star density, presumably to the eleventh magnitude stars, the extent in R.A. is apparently $4\frac{3}{4}^h$ to $7\frac{1}{4}^h$ (in reality it is $5^h\ 20^m$, as there is an extraordinary scarcity of faint stars during R.A. $4^h\ 0^m$ to 5^h, with a very precipitous rise—a perfect wall of stars—at $5^h\ 15^m$); thus, $6\frac{1}{4}^h$ against 2^h. The minimum is at 11^h and 13^h; there is a small yet distinct rise at 12^h, with an "accidental" minimum at 4^h, possibly a small or confined stellar vacuity and peculiar to the Oxford zones.

TABLE VII.

Dec.	Number of Plates.	Stars Measured.	Stars in a Square Degree.	Average Number of Stars each Plate.	Number measured *plus* the Extra Stars indicated in Table VI.	Stars in a Square Degree.
+25°	138	34141	52·6			
	180	47428	56·2	263·5	49215	57·8
+26	118	31877	57·4			
	180	60212	71·5	335·5	66776	78·9
+27	106	26545	53·3			
	180	53241	63·1	295·8	54984	65·1
+28	56	18103	68·8			
	160	52926	70·5	330·8	60216	80·2
+29	97	30853	67·6			
	160	53272	71·0	333·0	61474	81·8
+30	0	0	...			
	160	66900	89·2	418·1	83952	111·8
+31	0	0	...			
	160	65826	87·7	411·4	67650	89·9
Sum or Mean	513 / 1180	399805	72·3	338·8	444267	80·2

I wish to express my thanks to Mr. E. A. Gray, Mr. F. H. Scragg, and Mr. F. F. Lovegrove for assistance in collecting the data upon which the tables are based.

University Observatory, Oxford:
1904 *April* 7.

A New Cluster in Cygnus, with Right Ascensions and Declinations of 103 *Stars included in it.* By F. A. Bellamy.

1. At the end of last year I exposed a plate for the purpose of obtaining a photograph of the region of Dr. Max Wolf's new variable B.D. +37° 3876 (No. 59, 1903, *Cygni*) in order to measure the positions of stars surrounding it. The plate (2294) was exposed for 10^m and 16^m on 1903 October 9, and the adopted centre is +37°, 20^h 16^m: the hour angle was about 4^h west. The plate probably contains stars fainter than the 13th magnitude.

2. Whilst proceeding with the measurement of stars close to the variable I was impressed with the appearance of a well-defined and compact cluster of stars about $1\frac{1}{2}^m$ away and became interested in ascertaining its name. Upon reference to Dreyer's "New General Catalogue of Nebulæ and Clusters" (*Memoirs, R.A.S.*, vol. xlix. p. 1) I failed to find any mention made about it, nor could I find it referred to in his additional catalogue (*Memoirs, R.A.S.*, vol. li. p. 185). Such being the case it occurred to me that it was desirable to fix the stars' positions, and the results form the basis of this communication.

3. Without further description, it need only be said that the plate was exposed in the 13-inch astrographic telescope, which was moved in decl. about 45″ between the two exposures, that I measured both images in two positions of the plate 180° apart, using the glass-ruled scale in the microscope, the means of these four measures in each coordinate were carried to the fourth decimal—*i.e.* to a unit ·0001 = 0″·03. The calculations were made according to the formulæ published by Professor Turner.

4. The reference stars for the purpose of determining the plate constants have been selected from the Lund A.G.C. These positions have been brought to the epoch 1900·0 with precessions given in that catalogue, but no proper motions have been applied. The standard coordinates ξ and η referred to the plate centre +37°, 20^h 16^m, were computed for fifteen stars chosen in the SW, NW, NE, SE corners of the plate and about 45′ from the plate centre. If we put ξ' and η' equal to $\xi+13$ and $\eta+13$ respectively we shall obtain the coordinates referred to the corner of the réseau; they will always be positive, and are easily compared with the measured x and y. Putting $ax+by+c=x-\xi$ and $dx+ey+f=y-\eta$, the values of a, b, &c., were found to be

a	b	c	d	e	f
−·00068	+·00601	+·0863	−·00609	−·00073	−·0360

These are derived from the mean of the two exposures. It is not considered that any appreciable error is introduced in the present case by proceeding in this way rather than by treating the exposures separately, and a great deal of work is saved

without sacrificing accuracy. Reference may be made to *Monthly Notices*, vol. lxiii. pp. 513, 514.

5. The plate constants were applied to the mean measured x and y, and the standard coordinates ξ' and η' are thus obtained; they are given in columns 7 and 8 of Table I. By means of tables existing in MS. at the Observatory these ξ''s and η''s were readily converted to differences in R.A. and Decl., and when applied to the adopted plate centre ($+37°$, $20^h\ 16^m$) the R.A.s and Decls. for 1900·0, as given in columns 9 and 10, are obtained.

6. The magnitudes given in columns 5 and 6 consist, in the former case, of the *sum* of the four measured diameters; and in the next column I have given the approximate magnitude in the usual notation. These were derived by plotting the B.D. mags., comparing them with the measured diameters, assuming that the sum of the four measures of the smallest star visible was equal to 10 units (13th mag.), and then drawing a curve through these points. They must be considered as quite approximate, though I think they are not more than half a magnitude in error.

Grouping them, I find there are two stars brighter than 8·0, six stars from 8·0 to 9·9 (inclusive), fifty-one from 10·0 to 11·9, and forty-four from 12·0 to 13·0.

Table I.

Oxford Ref. No.	Potsd. No. on Plate 922.	B.D. $+37°$ No.	Magnitude. B.D.	Magnitude. Oxford Diameter (sum of 4).	Magnitude. Inferred.	$\xi + 13'\cdot0000$. 1900·0. R.I.	$\eta + 13'\cdot0000$. 1900·0. R.I.	Deduced R.A. 1900·0. 20^h+ m s	Deduced N. Dec. 1900·0. $37°+$ ′ ″
1	...	...	...	27	12·3	3·8339	17·4099	12 9·37	21 49·0
2	...	...	...	13	12·9	3·9234	15·2629	12 12·15	11 5·2
3	...	...	...	23	12·5	3·9586	15·7206	12 12·92	13 22·7
4	...	...	...	28	12·3	3·9855	17·9992	12 13·03	24 46·2
5	...	...	...	46	11·6	4·0044	17·0919	12 13·73	20 14·1
6	212	...	...	57	11·1	4·0224	15·4606	12 14·59	12 4·8
7	...	...	...	19	12·6	4·0989	16·8512	12 16·17	19 2·2
8	...	...	...	19	12·6	4·0949	15·8247	12 16·32	13 54·3
9	222	...	...	80	10·2	4·2649	16·0572	12 20·53	15 4·5
10	...	...	...	25	12·4	4·3602	16·8854	12 22·73	19 13·2
11	...	...	...	28	12·3	4·3768	15·6205	12 23·45	12 53·8
12	...	...	...	48	11·5	4·3964	17·3520	12 23·53	21 33·2
13	225	3855	9·3	117	8·8	4·3961	16·9167	12 23·62	19 22·7
14	...	...	...	18	12·7	4·3956	16·6273	12 23·68	17 55·9
15	231	...	...	80	10·2	4·5084	17·9302	12 26·21	24 27·1

Oxford Ref. No.	Potsd. No. on Plate 922.	B.D. +37° No.	Magnitude. B.D.	Magnitude. Oxford Diameter (sum of 4).	Magnitude. Inferred.	ξ+13°0000. 1900·0. R.I.	η+13°0000. 1900·0. R.I.	Deduced. R.A. 1900·0. 20h+ m	s	Deduced. N. Dec. 1900·0. 37°+ ′	″
16	230	3856	9·5	85	10·0	4·5002	16·0264	12	26·45	14	55·9
17	...	...	...	29	12·2	4·5054	16·3780	12	26·50	16	41·4
18	...	...	...	25	12·4	4·5141	16·3450	12	26·72	16	31·5
19	...	...	...	45	11·6	4·5641	16·3923	12	27·97	16	45·8
20	...	...	...	44	11·6	4·5854	17·7116	12	28·20	23	21·6
21	...	...	...	37	11·9	4·6453	15·4176	12	30·24	11	53·7
22	238	...	...	57	11·1	4·6625	15·8728	12	30·56	14	10·3
23	...	...	...	34	12·0	4·7154	17·7746	12	31·45	23	40·9
24	...	...	...	41	11·8	4·7196	15·5316	12	32·08	12	28·1
25	...	...	...	43	11·7	4·7503	16·0329	12	32·73	14	58·6
26	...	...	...	31	12·2	4·9068	16·7792	12	36·49	18	42·8
27	...	...	...	39	11·8	4·9245	15·3821	12	37·25	11	43·8
28	...	...	...	52	11·4	5·0770	17·3378	12	40·65	21	30·8
29	...	...	...	26	12·4	5·1042	17·5352	12	41·30	22	30·1
30	259	...	...	62	10·9	5·2228	16·3553	12	44·53	16	36·5
31	260	...	...	53	11·3	5·2448	16·6291	12	45·03	17	58·7
32	...	...	...	27	12·3	5·2506	16·6810	12	45·16	18	14·3
33	...	...	...	29	12·2	5·2931	16·8770	12	46·19	19	13·2
34	263	...	...	54	11·3	5·3090	15·8604	12	46·80	14	8·3
35	...	...	...	(13)	12·4	5·3197	16·9427	12	46·84	19	33·0
36	264	...	...	52	11·4	5·3333	17·1075	12	47·15	20	22·4
37	...	...	...	27	12·3	5·3392	16·7315	12	47·38	18	29·7
38	...	...	...	33	12·1	5·3524	16·8209	12	47·69	18	56·5
39	267	3859	9·2	105	9·2	5·3645	17·0508	12	47·95	20	5·5
40	268	...	...	45	11·6	5·3706	17·0289	12	48·10	19	59·0
41	271	...	...	51	11·4	5·4001	16·8110	12	48·89	18	53·6
42	274	...	...	67	10·7	5·4220	17·3410	12	49·33	21	32·7
43	275	3860	8·0	143	7·8	5·4246	17·0366	12	49 46	20	1·4
44	276	...	...	40	11·8	5·4255	17·1031	12	49·47	20	21·3
45	279	3862	9·2	118	8·8	5·4674	17·0731	12	50·53	20	12·5
46	...	...	...	35	12·0	5·4897	17·4039	12	51·02	21	51·8
47	...	...	...	(19)	11·9	5·4861	16·9266	12	51·03	19	28·6
48	...	...	...	37	11·9	5·4980	17·2611	12	51·26	21	8·9

Oxford Ref. No.	Potsd. No. on Plate 922.	B.D. +37° No.	Magnitude. B.D.	Magnitude. Oxford Diameter (sum of 4).	Magnitude. Inferred.	ξ + 13'0000. 1900'0.	η + 13'00000. 1900'0.	Deduced. R.A. 1900'0.	Deduced. N. Dec. 1900'0.
						R.I.	R.I.	20^h + m s	37° + ′ ″
49	282	...	...	61	11·0	5·4944	15·9543	12 51·44	14 36·9
50	280	...	...	54	11·3	5·5057	17·0584	12 51·50	20 8·2
51	283	...	...	67	10·7	5·5262	17·0414	12 52·01	20 3·1
52	...	...	...	46	11·5	5·5342	16·9546	12 52·23	19 37·1
53	...	...	...	41	11·8	5·5498	17·3140	12 52·55	21 24·9
54	...	...	...	30	12·2	5·5562	16·8713	12 52·80	19 12·2
55	289	...	...	54	11·3	5·5784	17·3640	12 53·26	21 40·0
56	...	...	...	44	11·6	5·5902	17·0413	12 53·62	20 3·2
57	...	...	...	24	12·4	5·5916	16·4699	12 53·78	17 11·9
58	290	...	...	50	11·4	5·5989	16·1612	12 54·02	15 39·3
59	...	...	...	20	12·6	5·6191	15·4738	12 54·67	12 13·1
60	...	...	...	43	11·7	5·6350	17·1721	12 54·72	20 42·6
61	...	...	...	21	12·6	5·6334	16·2634	12 54·87	16 10·0
62	292	...	...	115	8·9	5·6459	17·5726	12 54·92	22 42·8
63	...	...	...	24	12·4	5·6551	16·9066	12 55·28	19 23·0
64	...	...	...	34	12·0	5·6796	15·8092	12 56·12	13 53·9
65	...	...	...	32	12·2	5·7157	16·9144	12 56·81	19 25·5
66	...	...	...	28	12·3	5·7701	16·3175	12 58·29	16 26·6
67	...	...	...	48	11·5	5·7890	17·5987	12 58·52	22 50·9
68	...	...	...	28	12·3	5·7928	17·3080	12 58·67	21 23·7
69	299	3864	8·8	131	8·3	5·8017	15·7340	12 59·20	13 31·6
70	...	...	...	46	11·6	5·8250	16·7212	12 59·59	18 27·8
71	303	...	...	54	11·2	5·8706	17·1674	13 0·65	20 41·7
72	...	...	...	30	12·2	5·9288	16·7298	13 2·20	18 30·6
73	307	...	...	82	10·1	5·9729	17·4677	13 3·17	22 12·0
74	312	...	...	46	11·6	6·0329	17·6432	13 4·64	23 4·8
75	...	...	...	25	12·4	6·0530	18·0341	13 5·07	25 2·1
76	...	...	...	44	11·6	6·0470	16·9992	13 5·12	19 51·7
77	...	...	...	23	12·5	6·1495	17·1029	13 7·68	20 23·0
78	...	...	...	30	12·2	6·1585	16·8592	13 7·95	19 10·0
79	...	...	...	22	12·6	6·1816	16·9166	13 8·52	19 27·2
80	...	...	...	38	11·9	6·1965	16·4708	13 8·98	17 13·5

Oxford Ref. No.	Potsd. No. on Plate. 922.	B.D. +37° No.	Magnitude. B.D.	Oxford Diameter (sum of 4).	Inferred.	ξ +13°0000. 1900·0. R.I.	η +13°0000. 1900·0. R.I.	Deduced. R.A. 1900·0. 20h+ m	s	N. Dec. 1900·0. 37+° ′	″
81	...	...	...	46	11·6	6·2058	17·2562	13	9·07	21	9·1
82	...	...	...	22	12·5	6·2119	17·0270	13	9·26	20	0·4
83	...	...	...	25	12·4	6·2409	15·9445	13	10·19	14	35·8
84	318	3865	9·0	110	9·0	6·2395	15·5333	13	10·24	12	32·4
85	...	...	...	42	11·7	6·2657	16·9085	13	10·64	19	25·0
86	...	...	...	27	12·3	6·3026	16·9153	13	11·56	19	27·1
87	...	...	...	24	12·4	6·3633	17·8313	13	12·92	24	2·0
88	323	...	...	74	10·4	6·3688	17·8779	13	13·05	24	16·0
89	...	...	...	25	12·4	6·4080	15·9192	13	14·40	14	28·5
90	...	...	...	27	12·3	6·4228	17·6979	13	14·44	23	22·1
91	...	...	...	23	12·5	6·4487	17·3122	13	15·17	21	26·5
92	...	...	...	22	12·5	6·4607	17·4992	13	15·44	22	22·6
93	332	3866	8·0	165	7·0	6·5215	17·4903	13	16·97	22	20·1
94	...	...	...	43	11·7	6·5105	15·4744	13	17·05	12	15·3
95	...	...	...	38	11·9	6·5223	17·0303	13	17·07	20	2·1
96	...	...	...	22	12·5	6·5426	17·7584	13	17·45	23	40·5
97	335	...	...	59	11·0	6·5783	16·7143	13	18·53	18	27·5
98	...	...	...	46	11·6	6·6396	17·9095	13	19·86	24	26·1
99	...	...	...	34	12·0	6·6463	17·6439	13	20·08	23	6·4
100	336	...	...	55	11·2	6·6446	16·3118	13	20·27	16	26·9
101	...	...	...	49	11·4	6·7036	16·9905	13	21·64	19	50·5
102	...	...	...	46	11·6	6·7046	17·0518	13	21·65	20	8·9
103	338	...	...	48	11·5	6·7990	16·6175	13	24·10	17	58·9

Nos. 35 and 47.—The magnitude in column 5 for these two stars depends upon the first exposure only. The quantities given in columns 7 to 10 also depend upon the first exposure; in the former case the second image is completely involved in the first exposure of another star, in the latter case the second image is too doubtful to be measured.

It should be stated that these two stars have been corrected for plate constants, which had already been determined for another purpose, from the first exposure. But the values for a, b, d, e might have been used from the mean of the two exposures (§ 4), as there is a fairly close agreement between these two sets of constants. Of course c and f must differ.

a	b	c	d	e	f
−·00077	+·00605	+·0874	−·00618	−·00067	−·1114

7. After these stars had been measured and nearly the whole of the computations had been completed, I became acquainted

with the fact that the region containing this cluster had already been measured at Potsdam, and the results are given in *Photographische Himmelskarte*, Band II., page 221, plate 922, where twelve stars are indicated with an asterisk as forming a small cluster. Having reached this stage it seemed to me that it would be of interest to compare these results with those from the Oxford plate 2294.

8. The plate centre of Potsdam plate 922, exposed in 1896 July 13 by Dr. Clemens, when it was 7^m west of the meridian, is given as $20^h\ 14^m\ 47^s{\cdot}2$, $+37°\ 0'\ 30''$, but the x and y coordinates given have not been corrected for plate constants, and none are indicated. To make the results comparable with those I have deduced from Oxford plate 2294 I have computed the positions of fifteen stars from the Lund A.G.C., adopting $+37°\ 0'\ 0''$, $20^h\ 15^m\ 0^s$ as a convenient centre, and, after a comparison of these places, reduced to coordinates ξ' and η' (1900·0), with the Potsdam measured x and y have solved the equations and obtained the following constants :

a	b	c	d	e	f
−·00001	−·00014	+·5125	+·00027	+·00004	−·1009

These corrections have been applied to the measures (as printed) in a similar manner as was performed for the Oxford measures (§§ 4 and 5), and the results (ξ' and η') transformed to differences of R.A. and decl. from the adopted plate centre ($+37°\ 0'\ 0''$, $20^h\ 15^m\ 0^s$). A comparison of the plates is given in Table II.

9. In this table I have only given the number, R.A., and decl. for Oxford and Potsdam. The corrected Potsdam measures (X, Y) can be found by inference from the Oxford R.A.'s and decl.'s (centre $+37°$, $20^h\ 16^m$). In the fifth and eighth columns are the differences O−P, the means being $-^s{\cdot}002$ and $+0''{\cdot}59$, indicating a correction required to the constant f.

Owing to the difference between the centres of the two plates, only seven of the fourteen reference stars are common to the two plates.

TABLE II.

Oxford No.	Potsdam No.	Oxford. h m s	Potsdam. s	O−P s	Oxford. ° ′ ″	Potsdam. ″	O−P ″
6	212	20 12 14·59	14·57	+0·02	+37 12 4·8	4·3	+0·5
9	222	20·53	20·52	+·01	15 4·5	4·3	+0·2
13	225	23·62	23·60	+·02	19 22·7	22·5	+0·2
15	231	26·21	26·24	−·03	24 27·1	26·2	+0·9
16	230	26·45	26·41	+·04	14 55·9	55·5	+0·4
22	238	30·56	30·55	+·01	14 10·3	9·4	+0·9
30	259	44·53	44·54	−·02	16 36·5	36·1	+0·4
31	260	45·03	45·01	+·02	17 58·7	58·2	+0·5
34	263	46·80	46·81	−·01	14 8·3	7·7	+0·6

Oxford No.	Potsdam No.	Oxford.	Potsdam.	O−P	Oxford.	Potsdam.	O−P
		h m s	s	s	° ′ ″	″	″
36	264	20 12 47·15	47·15	·00	+37 20 22·4	22·1	+0·3
39	267	47·95	47·91	+·04	20 5·5	4·7	+0·8
40	268	48·10	48·04	+·06	19 59·0	60·8	−1·8
41	271	48·89	48·90	−·01	18 53·6	52·9	+0·7
42	274	49·33	49·34	−·01	21 32·7	31·5	+1·2
43	275	49·46	49·42	+·04	20 1·4	1·1	+0·3
44	276	49·47	49·50	−·03	20 21·3	20·8	+0·5
...	277	...	49·59	...	19 ...	55·2	...
45	279	50·53	50·50	+·03	20 12·5	11·8	+0·7
49	282	51·44	51·52	−·08	14 36·9	37·4	−0·5
50	280	51·50	51·50	·00	20 8·2	7·3	+0·9
51	283	52·01	52·03	−·02	20 3·1	2·6	+0·5
55	289	53·26	53·33	−·07	21 40·0	39·5	+0·5
58	290	54·02	54·08	−·06	15 39·3	38·8	+0·5
62	292	54·92	54·88	+·04	22 42·8	41·6	+1·2
69	299	59·20	59·18	+·02	13 31·6	31·3	+0·3
71	303	13 0·65	0·66	−·01	20 41·7	40·8	+0·9
73	307	3·17	3·19	−·02	22 12·0	11·2	+0·8
74	312	4·64	4·66	−·02	23 4·8	3·6	+1·2
84	318	10·24	10·19	+·05	12 32·4	30·6	+1·8
88	323	13·05	13·08	−·03	24 16·0	14·9	+1·1
93	332	16·97	16·93	+·04	22 20·1	19·4	+0·7
97	335	18·53	18·57	−·04	18 27·5	27·0	+0·5
100	336	20·27	20·27	·00	16 26·9	26·4	+0·5
103	338	24·10	24·15	−·05	17 58·9	58·6	+0·3
			Mean	−·002		Mean	+0·56

Potsdam 277 has a double note of interrogation against the printed measures. I cannot find anything on the Oxford plate to agree with this, but on the original measures I have the following note written opposite to Oxford No. 43: "? another star coalesced in the image of the bright star in the position *s.f.*" It is therefore very probable that Potsdam 277 is a star, as the position given would agree with my note: the semi-diameter of the image would be quite sufficient to overspread the faint star.

Potsdam 268 (Oxford 40). On the Oxford plate the images are partly involved in those of No. 39.

10. At the moment of completing this paper an advance copy of the first volume of the Greenwich Astrographic Catalogue was received by Professor Turner. In view of what has just been written in § 6 about the magnitudes it will be of interest to apply the Astronomer Royal's method for converting photographic into actual magnitudes to the measured diameters given

for this plate (2294) in column 5 ; it will, however, be necessary to divide those sums by 2 to reduce them to the same scale as adopted at Greenwich, *i.e.* the sum of D and R diameters.

In the formula given by Dr. Christie

$$\text{Mag.} = C - n\sqrt{d}$$

where C is a constant to be determined, d is the measured diameter, the unit being 0″·15 ; and n a constant which varies slightly with the duration of exposure. The mean value of n is found to be at Greenwich 0·73 for 20 sec., 0·77 for 6 min., and 0·84 for 40-min. exposures. In making the present comparison I have adopted 0·82 as the value of n for plate 2294, and have used B.D. magnitudes for twenty-two stars in the comparison with the Oxford measures, when divided by 2, and obtain 14·78 as the value of C. From this data I have formed Table III.

Table III.

Diameter.	$0{\cdot}82\sqrt{d}$.	$C-0{\cdot}82\sqrt{d}$. mag.	Diameter.	$0{\cdot}82\sqrt{d}$.	$C-0{\cdot}82\sqrt{d}$. mag.
5	1·84	12·94	40	5·18	9·60
10	2·59	12·19	50	5·80	8·98
15	3·17	11·61	60	6·36	8·42
20	3·67	11·11	70	6·86	7·92
25	4·10	10·68	80	7·33	7·45
30	4·49	10·29	90	7·78	7·00
35	4·85	9·93			

University Observatory, Oxford :
1904 *May* 12.

The Definitive Places of the Standard Stars for the Northern Zones of the Astronomische Gesellschaft. By A. M. W. Downing, D.Sc., F.R.S.

In *Astron. Nachrichten*, Nos. 3927–9, Professor Auwers has published his definitive corrections to the provisional places of the standard stars for the northern zones of the *Astronomische Gesellschaft*, which have been adopted in the reduction of the German zone observations. The Appendix to the *Berliner Jahrbuch* for 1906 supplies the corresponding corrections, for 1906·0, to the places of these stars as given in the body of that work.

It may be of some interest, therefore, to exhibit the results of a comparison of the places of the *Berliner Jahrbuch* stars, thus corrected, with their places as derived from Newcomb's "Fundamental Catalogue," which are given in the *Nautical*

Almanac for 1906; thus furnishing the systematic differences between the two Fundamental Catalogues from the North Pole to −25° Decl. for the epoch 1906.

The stars available for the comparison have been arranged in order of declination, and then combined in convenient groups. The further comparison in R.A. of stars within 13° of the pole has not been carried out. In forming the groups the following stars have also been omitted as being unsuitable for the purpose, viz. 61 *Cygni*, *α Canis Minoris*, *γ Virginis*, and *α Canis Majoris*.

The individual comparisons for these omitted stars, taken in the sense *Nautical Almanac*—*Berliner Jahrbuch*, are as follows:

Name of Star.	Approx. Decl.	Δα.	Δδ.
	° ′	s	″
λ Ursæ Minoris... ...	+89 0	−·436	...
α Ursæ Minoris... ...	88 48	−·083	...
51 H. Cephei	87 12	−·314	...
δ Ursæ Minoris... ...	86 37	−·028	...
ε Ursæ Minoris... ...	82 12	−·025	...
ζ Ursæ Minoris... ...	78 5	+·098	...
γ Cephei...	77 6	+·062	...
61′ Cygni	38 17	−·044	+·21
α Canis Minoris... ...	+ 5 28	−·003	−·66
γ′ Virginis	− 0 56	+·061	+·29
α Canis Majoris... ...	−16 35	−·093	+·11

The following Table exhibits the results for the groups, taken in the sense already mentioned, and the number of stars in each group. The total number of stars used for the comparison is in R.A. 255, and in Decl. 262.

Mean Decl.	Mean Δα.	Mean Δδ.	No. of Stars.	Mean Decl.	Mean Δα.	Mean Δδ.	No. of Stars.
° ′	s	″		° ′	s	″	
+87 54	...	+·04	4	22 34	+·005	−·03	17
79 8	...	−·04	3	17 26	+·006	+·13*	15
70 50	+·052	−·01	5	12 41	+·006	+·10	19
62 40	+·048	+·03	8	7 46	+·004	+·15	24
57 32	+·047	+·15	9	+ 3 8	+·002	+·28	12
52 40	+·019	+·17	5	− 1 58	+·003	+·28	18
47 41	+·011	+·11	6	7 56	+·001	+·17	19
42 45	+·017	+·15	9	12 21	+·002	+·21	10
38 8	+·006	+·04	9	16 50	−·002	+·24	17
32 25	+·001	+·01	12	22 31	−·001	+·10	15
27 41	+·005	+·05	16	−28 35	−·003	−·14	10

* Or +″·08 (adopted for curve) omitting *α Delphini*, for which Δδ = +″·86.

The above quantities have been graphically represented on cross-ruled paper, and curves drawn in the usual manner, from which the $\Delta\alpha$ and $\Delta\delta$, representing Newcomb—Auwers for the epoch 1906 have been read off for each 5° of declination. The results are exhibited in the columns headed I. in the last Table below. The proper quantities, interpolated from those given in these columns, have been then applied, with reversed signs, to the $\Delta\alpha$ and $\Delta\delta$ for each individual star (omitting the four stars mentioned above) which lies between the limits of Decl. +50° and −28°. These stars have been arranged in order of R.A., and the residual differences combined in groups of 1^h each. The results, which are exhibited in the following Table, are assumed to represent the systematic differences depending on R.A.

By proceeding exactly as before in the case of the differences depending on declination, those depending on R.A. have been read off from the appropriate curves for the beginning of each hour of right ascension. The results are exhibited in the next Table but one.

Mean R.A. h m	Mean $\Delta\alpha$. s	Mean $\Delta\delta$. "	No. of Stars.	Mean R.A. h m	Mean $\Delta\alpha$. s	Mean $\Delta\delta$. "	No. of Stars.
0 31	+·013	−·02	10	12 27	+·001	−·06	9
1 37	·000	·00	9	13 32	−·007	+·04	8
2 29	+·003	−·09	8	14 25	+·010	−·05	7
3 31	+·006	+·03	12	15 34	−·004	+·07	10
4 35	+·002	+·07	10	16 29	−·008	+·07	10
5 27	+·003	·00	17	17 24	−·002	−·05	9
6 32	+·001	+·08	10	18 33	−·011	+·01	8
7 28	−·001	·00	6	19 25	−·003	−·05	11
8 30	·000	+·02	9	20 29	+·003	·00*	11
9 33	−·008	+·02	7	21 28	·000	·00	7
10 26	−·003	+·09	6	22 30	+·001	−·01	11
11 20	·000	−·09	9	23 24	−·002	+·01	8

N−A (1906).

R.A. h	$\Delta\alpha_\alpha$. s	$\Delta\delta_\alpha$. "	R.A. h	$\Delta\alpha_\alpha$. s	$\Delta\delta_\alpha$. "
0	+·003	·00	7	·000	+·04
1	+·007	−·01	8	−·001	+·01
2	+·005	−·03	9	−·002	+·02
3	+·006	−·03	10	−·005	+·03
4	+·005	+·05	11	−·002	−·01
5	+·004	+·03	12	+·002	−·05
6	+·003	+·04	13	+·002	−·01

* Or −"·07 (adopted for curve) omitting α *Delphini*, for which $\Delta\delta$ = +"·76.

R.A. h	$\Delta\alpha_a$. s	$\Delta\delta_a$. "	R.A. h	$\Delta\alpha_a$. s	$\Delta\delta_a$. "
14	·000	·00	19	−·009	−·02
15	−·001	+·01	20	+·001	−·06
16	−·007	+·07	21	+·002	−·03
17	−·005	−·01	22	+·002	−·01
18	−·008	−·02	23	·000	−·01

The proper quantities, interpolated from the immediately preceding table, have then been applied, with reversed signs, to the differences for each individual star arranged in order of declination. The residuals give a second approximation to the systematic differences depending on declination, which (proceeding exactly as in the first approximation) are found to be those given in the columns headed II. in the final Table.

N−A (1906).

Dec. °	$\Delta\alpha\delta$. I. s	$\Delta\alpha\delta$. II. s	$\Delta\delta\delta$ I. "	$\Delta\delta\delta$ II. "
+85	...	...	+·01	+·01
80	...	...	−·03	−·04
75	...	...	−·03	−·03
70	+·052	+·053	−·01	·00
65	+·049	+·050	+·01	+·02
60	+·047	+·047	+·09	+·10
55	+·033	+·032	+·16	+·17
50	+·013	+·014	+·14	+·14
45	+·012	+·011	+·13	+·13
40	+·010	+·009	+·10	+·10
35	+·001	+·001	+·02	+·02
30	+·002	+·003	+·03	+·03
25	+·005	+·005	·00	−·01
20	+·005	+·004	·00	−·01
15	+·006	+·006	+·10	+·10
10	+·005	+·006	+·11	+·11
+ 5	+·003	+·002	+·22	+·22
0	+·002	+·003	+·28	+·27
− 5	+·002	+·002	+·20	+·21
10	+·001	·000	+·18	+·18
15	·000	−·001	+·23	+·23
20	−·001	−·001	+·18	+·18
−25	−·002	·000	·00	·00

It is very satisfactory to find a considerably closer agreement between the two systems of fundamental star places, due to the application of the definitive corrections deduced by Professor Auwers, than formerly obtained. The great improvement that has been effected in this respect is made obvious by comparing the reduction tables given above, for 1906, with those in Dr. Cohn's paper published in *Astron. Nachrichten*, No. 3742, which exhibit similar quantities applicable to the provisional fundamental catalogues of the *Astronomische Gesellschaft*, for the epochs 1875 and 1900. In particular it will be noticed that the right ascensions of stars of moderate declination are now in very close accord indeed; due not only to the agreement in the adopted position of the equinox in the respective systems, but also to the agreement (at the epoch considered) in the mean positions of groups of stars extending over a considerable range of declination.

H.M. Nautical Almanac Office:
1904 *May* 9.

Note on a Suggested Method of Determining the Declination of Stars. By A. E. Conrady.

The values of the declination of stars and of the latitude of observatories depend directly on the readings of graduated circles, and are limited to the accuracy with which the errors of graduation and of flexure have been, or can be, determined, besides involving errors of the refraction tables.

The following method of determining the declination of stars not too near the equator requires circle readings of a lower order of accuracy than that of the resulting declinations. I believe it is novel; whether it is of practical value remains an open question.

When the idea first occurred to me (1899 March 21) I began my note of it as follows:

"The clock is without doubt the most accurate and cheapest instrument for measuring angles; and in the case of many observations, made at different periods of the day and of the year, it is entirely free from constant errors.

"The question arises, whether declinations and latitude might not be obtained by transit observations based on observations of time only. This is indeed possible."

The method calls for a transit instrument in the meridian firmly united with a second telescope which has its optical axis parallel to the plane defined by the axis of rotation and the optical axis of the principal telescope, and is capable of being

fixed in any direction intermediate between that of the axis of rotation and that of the principal telescope.

When the principal telescope describes the meridian the auxiliary telescope describes a small circle parallel to the meridian. Then the angular distance between the two telescopes, for which the symbol C seems appropriate (for it really represents a very large intentional collimation error), is connected with the hour-angle t, at which a star of declination δ passes the small circle by the equation

$$\sin C = \sin t \cos \delta \quad \ldots \quad \ldots \quad (1)$$

For stars near the equator $\cos \delta$ differs but slightly from unity, and an error in the assumed value of δ has but very little effect on the resulting value of C; hence C may be determined by transits of equatorial stars without requiring an accurate knowledge of their declination, and if the stars are also observed in the principal telescope, errors in the tabular right ascension, as well as small errors in the position of the instrument, will also be eliminated.

C being thus determined, the telescopes are directed towards the pole and six-hour circle, when stars of declination nearly equal to C can be observed either by transit or, preferably, by micrometrical bisections in the manner which is practised with the transit instrument in the prime vertical. A transposition of equation (1) then gives the declination as

$$\cos \delta = \frac{\sin C}{\sin t} \quad \ldots \quad \ldots \quad (2)$$

and as t is now nearly 90° this observation is practically free from errors in the time or in the R.A. of the observed stars. The principal telescope will, of course, be used in both positions to control the instrumental constants.

It would at first sight seem that these observations would be affected by refraction even more strongly than measurements with the meridian circle, but in reality the refraction is to a very great extent eliminated; for if we consider the triangle between zenith, star, and the spherical perpendicular C, calling the angle at the star p, we shall have the differential relation

$$\partial C = \partial \zeta \cos p$$

$\partial \zeta$, being the refraction, is very nearly equal to a constant R multiplied by the tangent of the zenith distance, or

$$\partial C = R \operatorname{tg} \zeta \cos p.$$

But the same triangle also gives

$$\cos p = \frac{\operatorname{tg} C}{\operatorname{tg} \zeta}$$

and, introducing this, we get

$$\partial C = R \operatorname{tg} C \quad \ldots \quad \ldots \quad (3)$$

that is to say, the refraction uniformly raises the small circle without interfering with its parallelism to the corresponding vertical great circle. Therefore the effect of refraction in these observations is limited, besides second-order terms, which are amenable to computation, to the comparatively slight departures from the tangent law and to the effects of changes in the density of the atmosphere, both of which are more accurately known within the narrow limits which come into question than the total amount of the refraction for similar zenith distances.

Flexure of the instrument next suggests itself as a source of error. This also will be eliminated to a very great extent if, after observing the equatorial stars, the instrument is reversed in its bearings for the observations on the six-hour circle, as the instrument will then remain very nearly in the same position with regard to the direction of gravity.

It is obvious that the method could be most advantageously used in an observatory on the 45th parallel, where the elimination of errors would be most complete, and that it could not be used at all either near the equator or near the poles.

The declination of suitable stars having been determined, the latitude of the place of observation could be obtained by the zenith telescope or by a transit instrument in the prime vertical, again without involving close readings of graduated circles.

It would seem that this method could be advantageously carried out photographically, photographs of equatorial and six-hour regions being taken on the same plate, the resulting star traces forming approximately right angles with each other and being thus clearly distinguished.

Bedford Park, W.: 1904 *May* 10.

Micrometrical Measures of Double Stars made with the $17\frac{1}{4}$*-inch Reflector. Second Series.* By the Rev. T. E. Espin, M.A.

In the following list Column 1 gives the number in Σ, OΣ, &c., Column 2 the approximate R.A. and decl. for 1880, Column 3 the position angle, Column 4 the distance, Column 5 the number of nights, Column 6 the magnitudes, Column 7 the date, and Column 8 any notes. In addition to the usual symbols, A.G. refers to stars found double in the Catalogue of the *Astronomische Gesellschaft.* Under the heading Various Stars will be found several new pairs that have been detected since the last list was published, or have been accidentally omitted.

Σ *Stars.*

Σ	R.A. 1880 h m	Decl. ° ′	P. °	D. ″	Nights.	Mags.	Date.	Notes.
38	0 28·7	+58 0	145·4	17·25	1	...	04·06	
70	46·7	+52 2	244·6	7·91	1	6·5, 9·4	01·89	
162	1 41·8	+47 18	205·1	1·58	1	...	00·99	AB
			179·6	20·16	1	...	00·99	AC
163	42·6	+64 15	33·8	34·47	2	6·0, 8·3	03·43	
182	47·9	+60 42	304·9	3·38	2	...	04·06	*Comes* 13 mag. 71° ± 32″ ±
230	2 6·5	+57 55	258·2	23·49	2	...	03·87	
272	24·4	+57 56	38·9	2·07	1	8·0, 8·0	03·88	
392	3 1·4	+52 29	346·1	25·09	1	...	01·00	
550	4 22·5	+53 36	306·7	10·24	2	...	01·58	1 Cameli
587	38·5	+52 54	183·6	21·04	2	...	01·51	
603	45·1	+49 23	239·2	8·28	1	...	01·04	
617	52·3	62 50	122·5	12·76	3	8·5, 9·0	04·05	
618	52·5	62 54	210·6	32·44	3	...	04·05	
718	5 23·1	49 19	76·0	7·69	1	6·5, 6·8	99·88	
865	6 4·9	51 12	66·9	5·28	1	...	01·04	
884	11·5	47 11	271·2	9·17	1	8·3, 8·5	01·07	
946	34·2	59 34	129·7	4·34	2	...	04·07	
948	35·6	59 34	112·8	1·44	2	...	04·07	AB 12 Lyncis
			305·9	8·67	2	...	04·07	AC
958	38·2	+55 50	256·2	5·16	2	6·0, 6·0	01·56	
1006	56·0	+62 56	71·8	29·83	1	...	04·05	
1091	7 20·8	+50 13	330·3	28·51	1	...	00·11	
1174	56·1	+47 36	215·0	5·40	1	...	01·11	
1184	8 1·5	+38 14	340·8	26·50	1	8·0, 8·3	00·10	*Comes* 14 mag. 18″ ± 210° ±
1217	15·9	+45 21	241·7	29·07	1	...	00·11	
1263	37·3	+42 10	21·1	53·75	1	...	00·11	
1346	9 17·1	+54 32	312·6	5·45	2	...	01·17	21 Ursæ
1815	14 7·9	+45 45	152·4	8·84	1	..	01·38	
1830	11·9	+57 13	292·1	7·02	2	8·5, 9·0	02·33	AB
			80·5	35·84	2	C = 11·8	02·33	AC
1831	12·3	+57 15	140·3	5·84	2	6·5, 8·5	02·33	
1982	15 45·8	+43 8	301·7	5·42	2	8·9, 9·1	01·44	
2006	58·0	+59 15	194·3	1·42	2	7·5, 9·3	00·96	AB
			218·8	45·04	2	C = 8·7	00·96	AC
2047	16 19·7	+47 55	331·5	2·24	1	...	00·40	
2054	22·2	+61 57	356·2	1·37	4	6·8, 7·2	03·60	

Σ	R.A. 1880 h m	Decl. ° ′	P. °	D. ″	Nights.	Mags.	Date.	Notes.
2065	28·5	+40 14	217·1	31·00	1	8·1, 8·6	00·45	
2092	37·4	60 56	2·8	8·49	3	7·8, 8·4	03·57	
2116	54·3	63 42	4·6	19·52	2	8·5, 9·0	03·61	
2124	58·8	65 22	87·1	15·01	2	8·5, 9·0	03·61	
2155	17 14·6	60 50	112·3	9·75	2	6·5, 8·9	03·56	
2180	26·5	50 57	262·9	3·30	2	...	02·57	
2199	36·4	55 48	84·3	1·58	3	7·5, 7·9	02·93	
2218	39·5	63 43	345·2	1·72	3	6·6, 7·8	03·57	
2219	40·1	61 39	99·8	17·61	2	7·6, 8·5	03·56	
2225	41·9	51 59	313·0	8·82	1	9·0, 9·2	03·69	
2273	58·6	64 9	284·1	20·45	2	6·5, 7·0	03·56	
			266·4	24·1	2	C = 12	03·56	(New)
2357	18 32·1	63 37	268·8	4·87	1	...	03·56	
2368	36·1	52 14	326·8	1·74	2	...	03·70	
2420	49·4	59 15	333·9	32·32	3	...	03·65	*o* Draconis
2440	56·1	62 14	122·2	16·78	2	6·5, 8·0	03·58	
2481	19 7·1	38 34	221·6	4·32	1	7·2, 7·4	00·52	
2487	9·7	38 55	83·1	28·13	1	...	00·52	*η* Lyræ
2554	32·1	59 59	195·4	17·97	2	8·3, 8·6	03·62	
2573	38·3	60 12	27·2	18·13	1	...	03·60	
2604	51·5	63 49	184·3	27·38	4	6·5, 8·5	02·97	
2611	55·2	47 2	206·0	5·40	1	8·1, 8·1	01·64	Both very yellow
2640	20 3·2	63 32	19·6	5·18	3	7·0, 10·0	03·88	
2691	25·2	37 43	32·3	17·22	2	8·0, 8·2	01·57	
2764	21 2·9	61 40	302·2	6·67	2	8·0, 8·3	03·88	
2846	50·1	45 12	266·4	3·32	2	...	01·76	
2896	22 14·6	62 46	241·8	20·95	2	...	02·87	
2906	21·4	36 50	1·1	3·86	1	6·7, 9·2	00·62	
2985	23 4·4	47 18	253·3	15·57	1	6·3, 6·9	01·81	Yellow; yellowish white
3049	52·9	55 6	325·9	3·05	3	...	03·80	*σ* Cassiopeiæ
3053	56·4	65 26	70·5	14·69	2	...	03·78	

OΣ *Stars.*

59	3 32·3	45 38	350·0	2·72	1	8·0, 8·7	00·11	
158 rej.	6 43·9	51 41	303·5	16·39	1	7·0, 12·5	01·04	
291 rej.	14 56·5	47 45	156·5	35·94	1	...	01·38	
312	16 22·3	61 46	141·5	5·25	3	...	03·60	*η* Draconis

Σ	R.A. 1880 h	m	Decl. °	′	P. °	D. ″	Nights.	Mags.	Date.	Notes.
329	17	20·1	37	3	13·6	32·75	3	7·3, 8·2	00·35	
366 rej.	19	9·8	34	1	230·6	21·28	1	7·5, 8·7	00·57	AB
					141·0	26·1	1	11·7	00·57	AC
					21·7	13·2	1	13·2	00·57	CD

OΣΣ *Stars.*

15	1	13·3	64	0	348·1	52·63	2	6·0, 7·5	03·35	Yellow white, pale red
26	2	11·0	59	29	199·9	63·06	1	...	03·86	
39	3	39·7	56	45	75·1	57·57	2	...	02·51	
44	4	8·6	45	53	321·8	58·49	1	...	01·06	
46	4	11·3	55	13	188·6	34·40	1	7·0, 12·0	01·07	AB (New)
...		...		...	160·2	98·88	1	C 7·2	01·07	AC
87	7	30·4	42	46	178·0	65·10	2	...	00·10	
156	17	29·9	55	14	312·1	61·84	2	...	03·66	ν^1 ν^2 Draconis
163		55·5	62	37	40·1	58·10	2	7·0, 7·1	02·69	
186	19	31·2	59	53	287·0	75·76	2	...	03·62	
194		51·3	59	23	359·6	75·37	1	...	03·64	
242	23	1·0	46	16	31·0	80·34	1	6·9, 7·5	01·81	
247		24·3	57	53	268·9	75·80	1	...	00·86	
...		...		...	254·9	10·92	1	11·0, 11·2	00·86	*h* 1889
...		...		...	114·5	42·79	1	6, 12		*h* 1888
248		40·1	50	0	339·8	22·94	2	6·5, 12·0	02·84	AB (New)
...		...		...	139·9	52·68	2	C = 9·1	02·84	AC
251		47·5	50	52	199·6	43·35	2	...	03·86	

h's *Stars.*

1066	0	59·0	62	0	297·3	7·32	3	8·6, 11·6	03·57	*h* 302·2
2069	1	35·8	52	41	240·4	20·11	2	9·2, 9·3	00·02	
1091		40·3	61	14	152·3	27·66	2	8·5, 8·9	04·05	
2085		41·9	52	12	60·8	10·18	2	9·3, 9·8	00·47	
2307	6	11·8	54	6	92·0	22·40	1	8·6, 11·5	00·12	
417 756	7	13·8	34	55	69·4	17·60	1	8·5, 13·5	00·10	
2478	8	55·1	56	9	199·4	17·96	2	9·6, 10·7	01·21	
1231	13	19·9	41	5	6·2	22·39	1	8·5, 12·0	01·39	
554	14	27·4	35	15	294·3	12·11	2	8·9, 9·1	02·33	
1261		50·7	58	3	16·3	9·13	2	9·2, 9·3	01·35	B.D. + 58°·1538
2771	15	9·2	54	27	279·7	37·88	1	8·8, 12·5	00·45	*h*'s R.A. 1^m too large

Σ	R.A. 1880 h	m	Decl. °	′	P. °	D. ″	Nights.	Mags.	Date.	Notes.
2784		24·7	50	3	229·4	17·28	1	9·0, 13·0	00·34	
587	16	35·7	37	46	300·5	16·00	1	8·8, 13·5	00·52	
2802		53·0	39	18	123·5	11·32	1	9·3, 14·0	00·54	
263		57·2	38	8	113·3	18·04	1	9·3, 11·5	00·52	
264	17	3·4	36	10	197·4	12·54	2	9·5, 12·0	00·56	
1342	18	40·3	43	19	185·7	18·80	1	9·0, 12·0	02·65	
1356		52·9	45	17	343·7	29·23	2	8·3, 9·0	01·63	
1357		53·6	45	37	212·1	26·23	2	7·9, 11·7	01·64	
1395	19	20·9	36	53	65·2	3·09	1	9·0, 9·2	01·55	
1439		42·3	55	33	199·6	30·21	2	7·0, 9·0	02·67	*h* 195°·0
2905		46·2	60	57	196·9	8·61	2	9·1, 11·6	02·69	
1457		53·4	37	36	44·2	11·58	1	9·1, 9·2	00·57	
1505	20	17·2	43	14	116·9	16·84	1	8·3, 10·5	00·53	*h* 110°·3
1580		44·2	55	26	246·8	10·70	1	8·9, 12·0	02·68	
1606		56·8	54	4	184·0	17·60	2	9·0, 11·0	03·19	
1654	21	25·5	61	5	30·8	3·98	3	9·1, 9·3	02·99	
1659		29·3	58	8	300·4	6·68	1	8·4, 9·0	00·62	*h* 312°·2
939		30·5	30	28	172·7	14·81	1	8·0, 12·0	00·71	
940		30·5	30	31	290·6	40·72	1	8·0, 10·0	00·71	
1673		34·5	43	48	82·8	5·76	1	8·3, 8·8	00·75	
1687		39·9	45	37	51·0	5·44	1	9·4, 9·9	00·75	*h* 238°·5
1697		44·9	34	18	260·4	8·86	2	8·0, 9·6	00·66	
3072		50·4	58	53	76·8	13·83	1	...	01·01	
1714		56·5	44	46	252·6	16·68	1	8·6, 8·8	00·75	
1788	22	29·6	40	57	300·3	3·18	1	8·4, 9·2	00·62	
1796		33·8	56	10	6·6	29·59	1	5·0, 11·0	02·83	*h* 22°·1
1808		41·2	48	25	317·4	10·06	1	9·5, 9·6	01·76	*h* 133°·1
1860	23	8·5	62	0	355·8	7·75	1	9·0, 11·5	04·02	*h* 14°·9
1876		20·0	36	10	211·6	10·12	1	9·4, 9·6	00·71	
1896		33·3	61	26	115·6	16·33	3	6·8, 11·2	02·77	

Various Stars.

Espin	0	16·4	53	40	191·7	10·23	3	8·4, 9·3	02·91	B.D. + 53°·54
Espin	3	28·1	63	49	171·9	3·71	3	9·9, 10·4	04·05	B.D. + 63°·435
Espin	4	12·7	59	20	58·9	32·14	1	6·0, 8·8	04·07	B.D. + 59°·793
Espin	5	18·5	62	35	222·9	2·32	1	8·7, 10·5	04·06	B.D. + 62°·756
Σ 961 rej.	6	39·9	41	15	48·6	8·04	1	9·2, 10·5	00·14	
A.G.		53·5	40	1	247·7	6·59	2	9·5, 11·5	00·51	BC (New)
					151·3	8·47	2	9·4, 9·5	00·51	AB B D. + 40°·1776

Σ	R.A. 1880 h m	Decl. ° ′	P. °	D. ″	Nights.	Mags.	Date.	Notes.
Σ 2069 rej.	16 31·7	34 5	71·3	27·27	1	6·8, 9·4	00·54	
KR. 47	17 51·7	64 16	204·9	8·10	2	8·9, 9·3	03·59	
Σ 2334 rej.	18 24·5	62 51	205·2	14·48	2	...	03·58	
Espin	31·5	58 41	260·0	13·98	2	8·9, 11·9	02·81	
Σ 2353 rej.	31·6	58 36	224·8	11·75	2	8·2, 13·7	02·81	
Σ 2365 rej.	34·4	63 36	23·7	19·71	4	7·5, 9·6	03·24	
Σ 2377 rej.	37·5	63 25	340·7	15·55	3	6·9, 9·8	03·14	
A.G.	19 9·4	47 34	228·9	5·24	2	8·5, 8·7	01·65	B.D. + 47°·2782
A.G.	44·7	51 36	258·4	13·47	3	8·4, 9·3	03·00	B.D. + 51°·2683
—	20 1	35 35	149·1	96·47	2	...	01·59	Sh. 314, Sh. 315
Espin	42·5	57 9	162·7	68·65	3	5·0, 8·7	02·83	B.D. + 57°·2240
A.G.	47·2	44 36	251·3	9·48	1	8·8, 9·1	00·75	B.D. + 44°·3603
A.G.	48·0	53 36	361·1	9·47	2	8·7, 8·8	02·73	AB B.D. + 53°·2506
			250·5	13·04	2	C = 12·5	02·73	AC
Σ 2772 rej.	21 5·5	53 52	230·5	11·44	1	...	00·75	
A.G.	14·2	38 39	112·8	5·78	2	8·4, 9·3	00·76	B.D. + 38°·4442
Holmes	16·6	58 11	244·2	12·75	2	9·0, 9·1	02·78	B.D. + 58°·2252
A.G.	27·0	44 37	181·8	4·08	1	9·0, 9·3	00·79	B.D. + 44°·3852
Σ 2808 rej.	30·5	30 28	321·6	19·12	...	...	00·71	
Σ 2844 rej.	48·6	64 20	258·5	11·16	2	8·0, 10·0	03·74	
Espin	22 51·9	64 9	330·5	2·76	1	11·0, 12·0	02·73	BC
			296·4	19·13	1	A = 10	02·73	AB (*h* 1833)
Espin	23 15·6	61 45	...	4 ±	...	11·5, 12·5	04·02	BC
				30 ±	...	A = 8·0	04·02	AB B.D. + 61°·2430

The Society has received the following circular letter from the Weather Bureau of the United States Department of Agriculture. The Council has directed that the letter should be published in the *Monthly Notices*.

U.S. Department of Agriculture:
Weather Bureau, Washington, D.C.
April 15, 1904.

SIR,—You may have noticed that records have been published by Messrs. C. Dufour, of Lausanne; H. H. Kimball, S. P. Langley, and C. G. Abbot, of Washington; Max Wolf, of Heidelberg; and Gorczynski, of Warsaw, showing that an appreciable general diminution of the transparency of the Earth's atmosphere took place some time during the year 1902, but disappeared at some time during 1903. This is an important

matter, and may possibly be made the basis of an explanation of other meteorological phenomena. I beg to ask whether you have any records that will assist in defining the dates of beginning and ending, and the extent of this change in transparency. Such records may consist of photometric or photographic observations of the brightness of the stars ; changes in the solar or stellar spectra ; unusual prevalence of halos, large Bishop's ring, or haze ; observations of heat received from the sun, as made with actinometers or pyrheliometers ; observations of the polarisation of the blue sky light and of scintillation of the stars.

Undoubtedly this diminution and increase of transparency began and ended at different dates in different places, as the phenomena spread gradually over the world during the years 1902 and 1903 ; additional records are therefore desired in order to trace its progress. Will you not kindly examine your records from this point of view and send me the result for publication in a general article on this subject ?

Very respectfully,

CLEVELAND ABBE,

Professor and Editor.

Errata.

Annual Report of the Council, page 333.

The Report of the Kodaikanal Observatory was communicated by Mr. C. P. Butler, who was in charge of the Observatory during the Director's absence in England.

Prof. Turner's Paper on Time of Sunset.

Page 193, line 18, *for* Belfast *read* Southport.